Stellar atmospheric photo-overshoot

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1 Introduction

Convection is a standard form of energy transport in regions of a star where the specific entropy decreases with height, i.e., the stratification is superadiabatic. Overshoot tends to occur on the boundaries between convective and radiative zones, and thus cannot occur in on its own. Overshoot is characterized by an oppositely oriented transport of energy, because overshooting plumes are heavier and cooler than their surroundings, so the product of velocity and temperature fluctuations is negative.

We report here the results of a numerical study of a stably stratified layer of gas whose opacity increases with decreasing pressure and therefore with height. Adiabatically upward moving blobs of fluid therefore become more opaque and can be elevated by radiation pressure. The elevated fluid becomes even more opaque and will be elevated even further. Likewise, downward moving fluid will become more transparent and will sink further, and can fill the place left by the upward moving fluid. This can lead to instability and the development of overturning motions, or even continuous mass loss. Similar effects also occur when the temperature dependence of the opacity has a local maximum, which can lead to turbulence in those layers where the temperature dependence of the opacity has a local maximum.

Our goal is to study basic properties of such flows in an idealized system. We perform a sequence of numerical experiments using a simple power law prescription for the opacity of the form

$$\kappa = \kappa_0 \, (\rho/\rho_0)^a \, (T/T_0)^b, . \tag{1}$$

where the exponents a and b and the pre-factor κ_0 will be varied, and the coefficients ρ_0 and T_0 will be kept fixed.

The numerical treatment of such a system poses numerical difficulties owing to the requirement of very short time steps. This is because the radiative cooling time becomes short at high temperatures, which is when the radiation pressure tends to play an important role. This was recently investigated in detail in a separate paper (Brandenburg & Das, 2019). To cope with this difficulty, we simply reduce the speed of light. This increases the effect of radiation pressure, but has no direct effect on the rest of the equations. However, since c is related to the Stefan-Boltzmann constant through $\sigma_{\rm SB} = a_{\rm rad} c/4$, there are two possibilities. Either we keep $a_{\rm rad}$ fixed and vary $\sigma_{\rm SB}$, or we vary $a_{\rm rad}$ and keep $\sigma_{\rm SB}$ fixed. In that case, however, it would therefore gives the system more radiation energy than it actually has.

2 The model

2.1 Governing equations

We solve the basic equations for the logarithmic density $\ln \rho$, the velocity $\boldsymbol{u}$, and the specific entropy s, in the form

$$\frac{\mathrm{D} \ln \rho}{\mathrm{D}t} = -\boldsymbol{\nabla} \cdot \boldsymbol{u}, \tag{2}$$

$$\rho \frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} = -\boldsymbol{\nabla} p + \rho \boldsymbol{g} + \frac{\rho\kappa}{c} \boldsymbol{F}_{\rm rad} + \boldsymbol{\nabla} \cdot (2\rho\nu \mathsf{S}), \tag{3}$$

$$\rho T \frac{\mathrm{D}s}{\mathrm{D}t} = -\boldsymbol{\nabla} \cdot \boldsymbol{F}_{\rm rad} + 2\rho\nu \mathsf{S}^2, \tag{4}$$

$$\hat{\boldsymbol{n}} \cdot \boldsymbol{\nabla} I = -\kappa\rho \, (I - S), \tag{5}$$

$$\boldsymbol{F}_{\rm rad} = \int_{4\pi} \hat{\boldsymbol{n}} I \, \mathrm{d}\Omega, \tag{6}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{F}_{\rm rad} = \int_{4\pi} (I - S) \, \mathrm{d}\Omega, \tag{7}$$

where $\boldsymbol{g} = (0, 0, -g)$ is the gravitational acceleration in Cartesian coordinates (x, y, z), c is the speed of light, $\boldsymbol{F}_{\rm rad}$ is the radiative flux, $\mathsf{S}_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i) - \frac{1}{3}\delta_{ij} \boldsymbol{\nabla} \cdot \boldsymbol{u}$ are the components of the traceless rate-of-strain tensor, and ν is the kinematic viscoity.

2.2 Nondimensional form of the equations

It is useful to write the equations in nondimensional form by normalizing temperature and density by some representative values that characterize the surface. In a non-convecting gray atmosphere with constant exponents a and b, the temperature falls of linearly with height until it reaches a constant value, T_0. In the deeper, optically thick part, the density falls off like a polytrope, so $\rho \propto T^n$ where $n = (3-b)/(1+a)$. In the upper optically thin part, which is isothermal, the density falls off exponentially with the pressure scale height $H_{\rm p0} = c_{\rm p} T \nabla_{\rm ad}/g$, which is also equal to the density scale height in this isothermal part. It is then convenient to measure lengths in units of $H_{\rm p0}$, i.e.,

$$[x] = H_{\rm p0}. \tag{8}$$

Since $g = $ const, it is convenient to measure time in units of $[t] = \sqrt{H_{\rm p0}/g}$, and velocity in units of

$$u_0 \equiv \sqrt{H_{\rm p0} g}. \tag{9}$$

Finally, we measure density and pressure in units of the values ρ_0 and p_0 at the crossover between polytropic and isothermal stratifications. They are related to each other through an equation of state

$$p_0/\rho_0 = \mathcal{R} T_0/\mu_0, \tag{10}$$

where $\mathcal{R}$ is the universal gas constant and μ_0 is the mean molecular weight.

To determine ρ_0, it is useful to recall the analytic solution for a gray atmosphere with constant a and b in the form (Brandenburg, 2016)

$$T/T_0 = \left[1 + (n+1)\nabla_{\rm rad}^{(0)} (p/p_0)^{1+a}\right]^{1/(4+a-b)} \tag{11}$$

with $\nabla_{\rm rad}^{(0)} = c_{\rm p} F_{\rm rad} \nabla_{\rm ad}/(gK_0)$ being the usual radiative double-logarithmic temperature gradient and $K_0 = 16\sigma T_0^3/(3\kappa_0\rho_0)$ the radiative conductivity evaluated for our representative values T_0 and ρ_0. Since $\nabla_{\rm rad}^{(0)}$ itself depends on ρ_0, we can determine ρ_0 such that $(n+1)\nabla_{\rm rad}^{(0)} = 1$. Using $F_{\rm rad} = \sigma_{\rm SB} T_{\rm eff}^4 = 2\sigma_{\rm SB} T_0^4$, where $T_{\rm eff}$ is the effective temperature, this yields

$$\rho_0 = 8/(3\kappa_0 H_{\rm p0}). \tag{12}$$

2.3 Stratification without radiation pressure

In thermodynamic equilibrium, the radiative flux must be constant, i.e.,

$$F_{\rm rad} = -K\, {\rm d}T/{\rm d}z = \text{const}, \tag{13}$$

where $K = 16\sigma_{\rm SB} T^3/(3\kappa\rho)$ is the radiative conductivity with $\sigma_{\rm SB}$ being the Stefan–Boltzmann constant, and z is the vertical coordinate in a Cartesian coordinate system. We have then a polytropic stratification with $\rho \propto T^n$, where

$$n = (3-b)/(1+a) \tag{14}$$

is the polytropic index.

The double-logarithmic temperature gradient is obtained by dividing the two equations through each other, i.e.,

$$\nabla = \frac{{\rm d}\ln \overline{T}}{{\rm d}\ln \overline{P}} = \frac{F\overline{P}}{K\overline{T}\,\overline{\rho} g} = \frac{F c_P \nabla_{\rm ad}}{Kg}, \tag{15}$$

where we have used the perfect gas equation of state in the form $\overline{P}/\overline{T}\,\overline{\rho} = c_P - c_V = c_P(1-1/\gamma) = c_P \nabla_{\rm ad}$. We can also define a *hypothetical* radiative temperature gradient $\nabla_{\rm rad}$ that would result if all the energy were carried by radiation, so we can write

$$F_{\rm tot} = \frac{Kg}{c_P \nabla_{\rm ad}}\, \nabla_{\rm rad}, \tag{16}$$

which follows from Equation (15).

Dividing Equation (13) by the equation for hydrostatic equilibrium, ${\rm d}P/{\rm d}z = -\rho g$, we have

$$\frac{{\rm d}T}{{\rm d}P} = \frac{F_{\rm rad}}{K_0 \rho_0 g} \frac{(P/P_0)^a}{(T/T_0)^{3+a-b}}, \tag{17}$$

where $K_0 = 16\sigma_{\rm SB} T_0^3/(3\kappa_0\rho_0)$ is a constant and $P/P_0 = (\rho/\rho_0)(T/T_0)$ is the ideal gas equation with a suitably defined constant $P_0 = (c_{\rm p} - c_{\rm v})\rho_0 T_0$. Here, ρ_0 and T_0 are reference values that were defined in Equation (1). Equation (17) can be integrated to give

$$(T/T_0)^{4+a-b} = (n+1)\nabla_{\rm rad}^{(0)} (P/P_0)^{1+a} + (T_{\rm top}/T_0)^{4+a-b}, \tag{18}$$

where $\nabla_{\rm rad}^{(0)} = F_{\rm rad} P_0/(K_0 T_0 \rho_0 g)$, which is defined analogously to the $\nabla_{\rm rad}$ without superscript (0) in Equations (15) and (16), and $T_{\rm top}$ is an integration constant that is specified such that $T \to T_{\rm top}$ as $P \to 0$. Note also that $4+a-b = (n+1)(1+a)$, where n was defined in Equation (14) as the

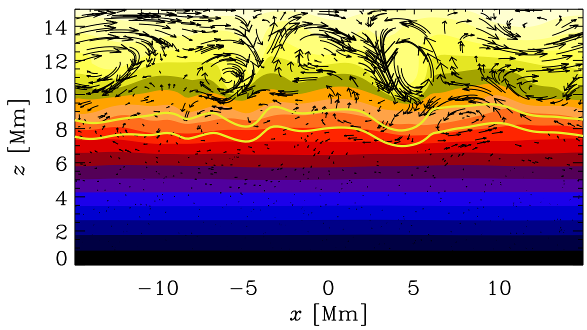

Figure 1: `pvar_s144x72a` The yellow horizontal line around $z \approx 8\,\mathrm{Mm}$ marks the loation of the $\tau = 1$ surface.

polytropic index, so the ratio of $4 + a - b$ to $1 + a$ is just $n + 1$, which enters in front of the $\nabla_{\rm rad}^{(0)}$ term in Equation (18). Since $K \propto T^{3-b}/\rho^{1+a} \propto T^{4+a-b}/P^{1+a}$, we have $K \rightarrow \mathrm{const} = K_0$ for $T \gg T_{\rm top}$.

(19)

Hot stars are observed to exhibit macroturbulence. Those stars have either no or only very shallow subsurface convection zones that are associated with local maxima in the temperature dependence of the opacity. Photoconvection is a form of convection that is driven by super-Eddington luminosities where the fluid remains suspended.

References

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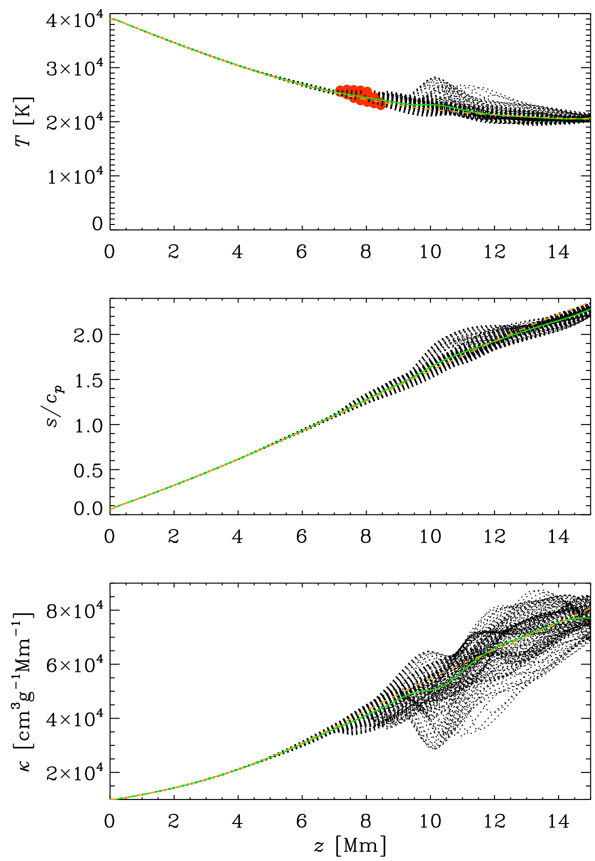

Figure 2: `ptt_s144x72a`