Radiative accretion disk models

Notes and calculations assembled by A. Brandenburg and U. Das

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1 Background

1.1 Cataclysmic variables

[The notes in this subsection are excerpts from an excellent wikipedia article[1].] Cataclysmic variables (CV) consist of a white dwarf (WD) primary and a mass transferring secondary (donor star). Strong UV and X-ray emission is often seen from the accretion disc, powered by the loss of gravitational potential energy from the infalling material. Material at the inner edge of disc falls onto the surface of the white dwarf primary. A classical nova outburst occurs when the density and temperature at the bottom of the accumulated hydrogen layer rise high enough to ignite runaway hydrogen fusion reactions. The accretion disc may be prone to an instability leading to dwarf nova outbursts, when the outer portion of the disc changes from a cool, dull mode to a hotter, brighter mode for a time, before reverting to the cool mode. Dwarf novae can recur on a timescale of days to decades.

1.2 Relevant literature

In a recent paper, Hirose et al. (2014) presented shearing box simulations of magneto-rotational instability (MRI) turbulence in a regime relevant to dwarf novae and soft X-ray transient outbursts. They find two stable thermal equilibria in the effective temperature versus surface mass density diagram, which is consistent with the hypothetical S-curve dependence adopted since the 1980s (Bath & Pringle, 1981, 1982; Meyer & Meyer-Hofmeister, 1981, 1982; Cannizzo et al., 1982). In their new work, Hirose et al. (2014) find that convection strengthens the dynamo and enhances the Shakura-Sunyaev α parameter (Shakura & Sunyaev, 1973) by generating vertical magnetic field that seed the axisymmetric MRI and by increasing cooling. Our goal is to provide an independent confirmation of the results of Hirose et al. (2014) and to arrive at a better understanding of the transition between the two branches that help parameterizing the relevant physics, which can then be used in simpler one-dimensional accretion disk models.

[1] `https://en.wikipedia.org/wiki/Cataclysmic_variable_star`

2 Polytropic solution

2.1 Hydrostatic balance

We expect accretion disks to be turbulent, e.g., because of MRI or because of convection in the outer layers. In both of these cases, the cause of turbulence is related to the occurrence of an instability of the hydrostatic base state, Once turbulence develops, the original base state would no longer be directly relevant, because it would become strongly modified. To understand this transition, we need to establish first the hydrostatic reference state.

Hydrostatic balance implies[2]

$$0 = -\boldsymbol{\nabla} h + T\boldsymbol{\nabla} S - \boldsymbol{\nabla}\phi, \tag{1}$$

where T is temperature, S is specific entropy, $h = c_\mathrm{p}T$ is the enthalpy for a perfect with fixed ionization with c_p being the specific heat at constant pressure, and $\phi = \frac{1}{2}(z_\infty - z)\,\Omega^2$ is the gravitational potential with z being the height from the midplane, z_∞ is the height of the surface, and Ω is the angular velocity at the position where the our local coordinate system is situated.

For a polytropic solution, the $T\boldsymbol{\nabla} S$ can be written underneath a gradient and can be combined with the enthalpy to give what we might call the pseudo-enthalpy, so

$$0 = -\boldsymbol{\nabla}(\tilde{h} + \phi) \tag{2}$$

so

$$\tilde{h} = \frac{1-\frac{1}{\gamma}}{1-\frac{1}{\Gamma}}\,c_pT = \frac{1}{2}\left(z_\infty^2 - z^2\right)\Omega^2, \tag{3}$$

[2] see the manual to the Pencil Code on `https://github.com/pencil-code`

where $\gamma = c_p/c_v$ is the ratio of specific heats, c_v is the specific heat at constant volume, and Γ is a parameter characterizing the polytropic solution and is related to the polytropic index; see Appendix A for details. For monatomic gas we have $\gamma = 5/3$. If we assume a Kramers-like opacity, then $n = 3.25$, so $\Gamma = 1+1/n = 1.308$; see Barekat & Brandenburg (2014) for details. Thus, we have

$$\tilde{h} = (n+1)\left(1-\frac{1}{\gamma}\right) c_p T = \frac{1}{2}\left(z_\infty^2 - z^2\right)\Omega^2, \quad (4)$$

and therefore

$$T = \tfrac{5}{17}\left(z_\infty^2 - z^2\right)\Omega^2/c_p. \quad (5)$$

2.2 Thermal balance

To sustain thermal equilibrium, we have to include a heating term $\mathcal{H}$, so the time-dependent entropy equation takes the form

$$\rho T \frac{\mathrm{D}s}{\mathrm{D}t} = \boldsymbol{\nabla}\cdot K\boldsymbol{\nabla}T + \mathcal{H} \quad (6)$$

Here we choose

$$\mathcal{H} = 5\times10^{-3}\,\mathrm{g\,cm^{-3}\,km^3\,s^{-3}\,Mm^{-1}}, \quad (7)$$

Using $\sigma_{\rm SB} = 5.67\times10^{-20}$ (in code units; see Barekat & Brandenburg, 2014), we have

$$\begin{aligned} T_{\rm surf} &= \left(\tfrac{3}{4} z_{\rm heat}\mathcal{H}/\sigma_{\rm SB}\right)^{1/4} \\ &= \left(0.75\times5\times10^{-3}/5.67\times10^{-20}\right)^{1/4} \\ &= 23980\,\mathrm{K}. \quad (8) \end{aligned}$$

We find

$$K = 7.14\times10^{-6} \quad (9)$$

For details about radiation transfer, see Barekat & Brandenburg (2014) and references therein. At some point, ionization will need to be included; see Bhat & Brandenburg (2016).

2.3 Sketchy notes

Tall box in Figure 3. Time step is less stringent when the opacity floor is allowed to be smaller, but it is quadratic in the invert mean free path, $\delta t \propto \ell^2 \propto (\kappa\rho)^{-2}$. Unclear.

2-D convection, $u_\downarrow = 1.2\,\mathrm{km\,s^{-1}}$, $u_\uparrow = 0.8\,\mathrm{km\,s^{-1}}$. Note the extreme aspect ratio.

nu	kappa0	omeg	Run	
5e-2	1e5		144f	
5e-2	3e5		144g	never ran
2e-2	1e5		288a	
1e-2	1e5	2.47	288b	

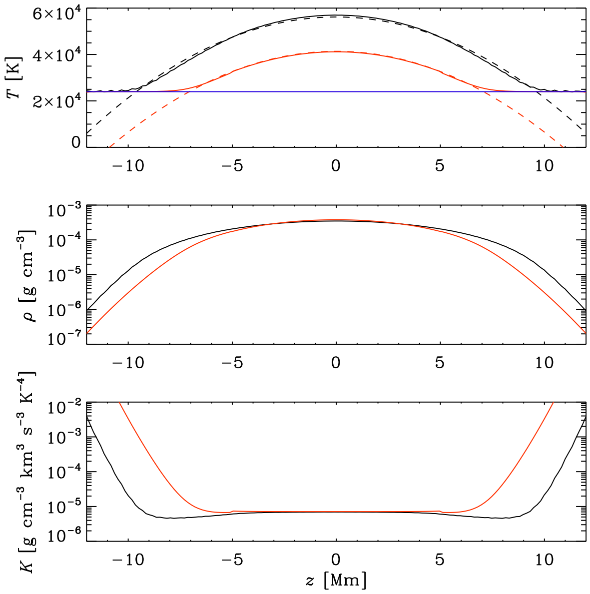

Figure 1: $\kappa_0 = 10^4$, $z_{\rm max} = 12$, $z_{\rm heat} = 5$, $\mathcal{H} = 5\times10^{-3}$ $a = 1$, $b = -3.5$. 144 meshpoints.

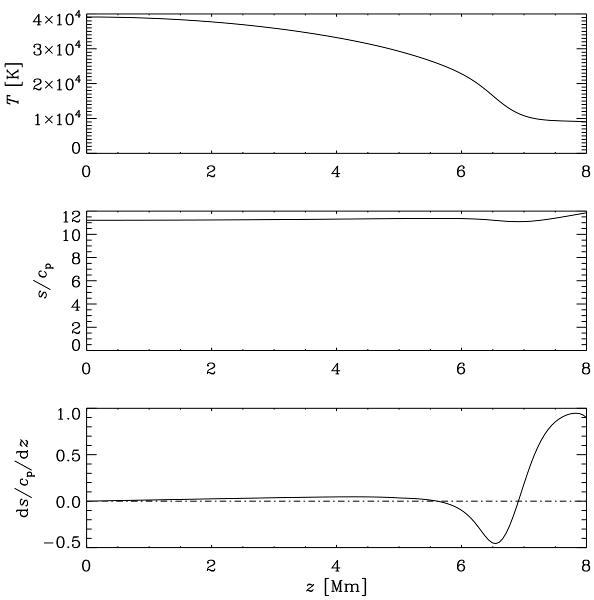

Figure 2: $\kappa_0 = 10^5$, $z_{\rm max} = 8$, $z_{\rm heat} = 5$, $\mathcal{H} = 10^{-4}$ $a_{\rm Kr} = 1$, $b_{\rm Kr} = -3.5$, $a_{\rm H}^- = 1$, $b_{\rm H}^- = 4$. 144 meshpoints.

2.4 New runs

$\Sigma = 2\times10^{-12}$, $\mathcal{H} = 7.1\times10^{-7}$, two stable disk states; see Figure 9.

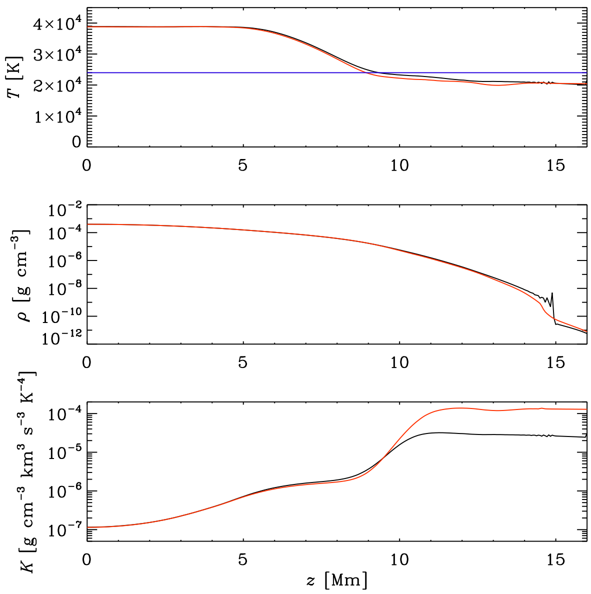

Figure 3: Floor values 0.2 and 0,02, kappa0=1e5 H=1e-4, time steps 4e-7 and 4e-6, respectively. 288 meshpoints, 1D, but time still short because of short timestep. Should plot s profile.

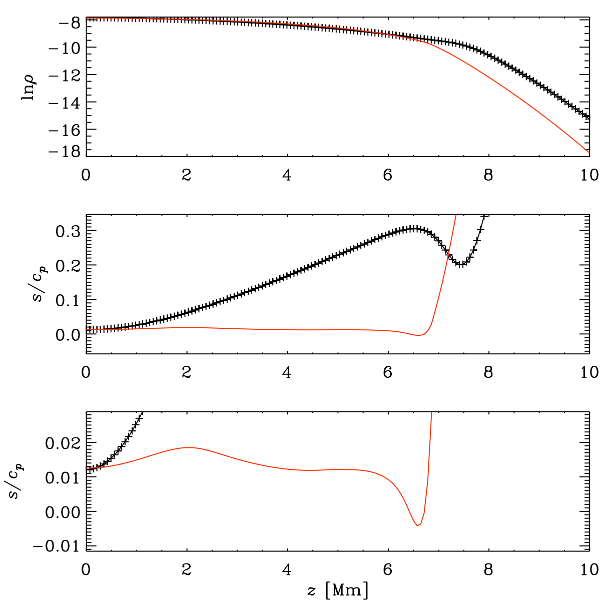

Figure 4: Stratification, 144f

3 CV disks

The following parameters have been assembled from the paper by Shaviv & Wehrse (1991).

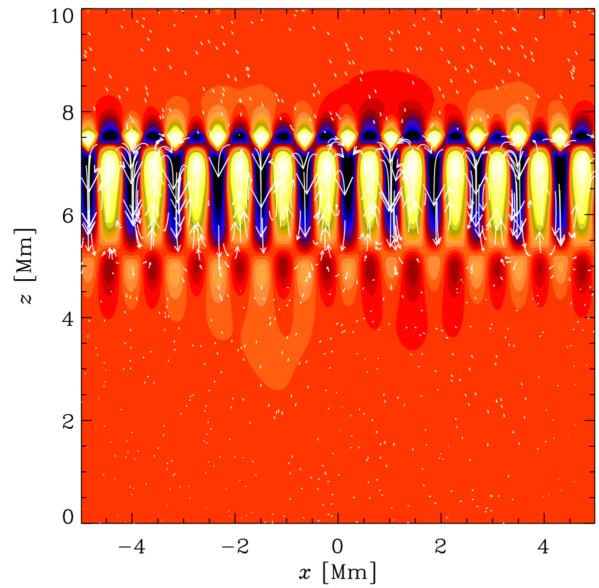

Figure 5: Vertical slice, 2D, 288x288, time step 2e-3. Initial amplitude 0.1 km/s. $t = 4\,\mathrm{ks}$. $\nu = 0.02\,\mathrm{Mm\,km\,s^{-1}}$.

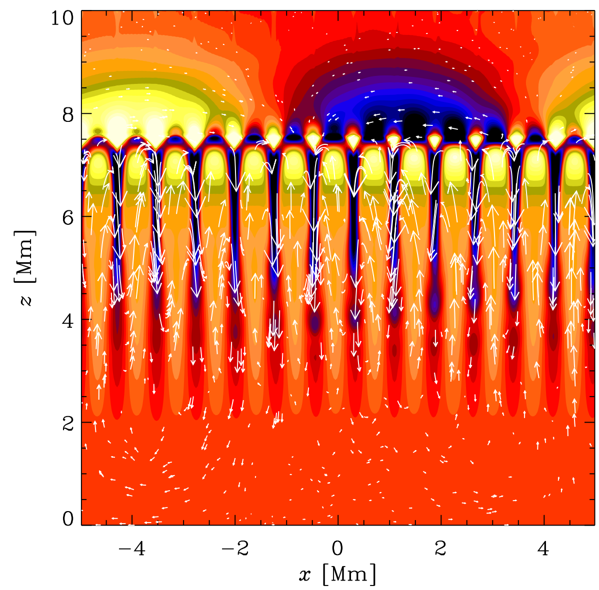

Figure 6: Vertical slice, 2D, 288x288, time step 8e-4. $t = 200\,\mathrm{ks}$ $\nu = 0.01\,\mathrm{Mm\,km\,s^{-1}}$. $\kappa_0 = 3 \times 10^5$, $\mathcal{H} = 10^{-4}$.

$M = 1\,M_\odot$, $R = 10^8\,\mathrm{m}$, so $\Omega = \sqrt{GM/R^3} = 12\,\mathrm{ks^{-1}}$, $\Sigma = 10^2\,\mathrm{g\,cm^{-2}} = 10^{-6}\,\mathrm{Mm\,g\,cm^{-3}}$.

We start with an isothermal model with

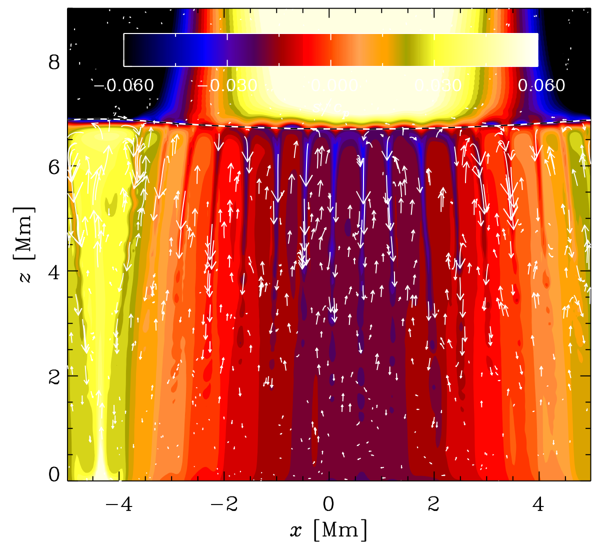

Figure 7: Vertical slice, 2D, 288x288, time step 1e-3. $t = 3600\,\mathrm{ks}$. $\nu = 0.005\,\mathrm{Mm\,km\,s^{-1}}$. $\kappa_0 = 10^6$, $\mathcal{H} = 10^{-5}$.

$c_\mathrm{s} = 36\,\mathrm{km\,s^{-1}}$, $\rho_0 = 10^{-6}\,\mathrm{G\,cm^{-3}}$.

```
kappa0 Heat ztau1 z [Mm]   taumax chimin run
1e10   1e-5 3.65  0 ... 4.5 289    3.2  CV288n
4e10   1e-5 5.37  0 ... 6.0 270    28   CV288q
1.6e11 5e-6 6.72  \pm 7.5   140   401   CV288q5

M_WD=0.8 Msun
R_WD=10 Mm
R=(6.67e-11*2e30/12e-3^2)^.3333333333333
0.9e8 = 90 Mm.
```

A Relation between Γ and n

A polytropic stratification is one where the stratifications of density and pressure, $\rho(z)$ and $P(z)$ are related to that of temperature $T(z)$ in a powerlaw fashion, i.e.,

$$\rho(z) \propto T(z)^n, \quad P(z) \propto T(z)^{n+1}. \tag{10}$$

where the latter relation is a consequence of the perfect gas equation, $P \propto \rho T$. Thus, $n = \mathrm{d}\ln\rho/\mathrm{d}\ln T$ and $n+1 = \mathrm{d}\ln P/\mathrm{d}\ln T$. It is also useful to define

$$\Gamma = \mathrm{d}\ln P/\mathrm{d}\ln\rho = \frac{n+1}{n} = 1 + \frac{1}{n}, \tag{11}$$

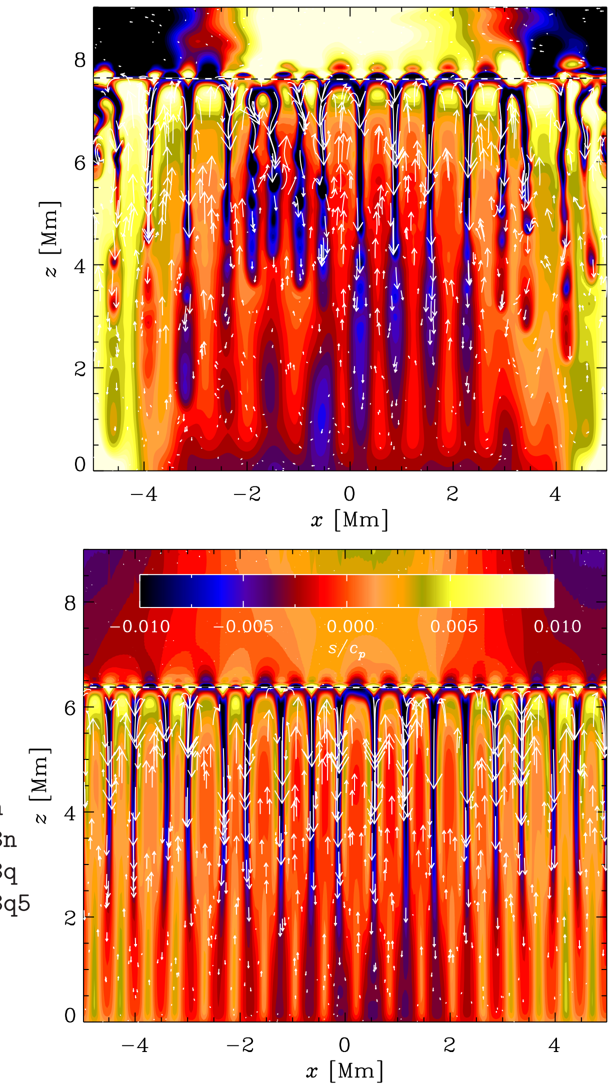

Figure 8: Vertical slice, 2D, 288x288, time step 8e-4. $t = 700\,\mathrm{ks}$, and $4200\,\mathrm{ks}$. $\nu = 0.01\,\mathrm{Mm\,km\,s^{-1}}$. $\kappa_0 = 10^6$, $\mathcal{H} = 10^{-5}$.

which is convenient as it can easily be compared with γ (which is a property of the gas, not of the stratification, as is *Gamma*). To see that, let us calculate the vertical gradient of the specific entropy s using the formula

$$\mathrm{d}s/c_\mathrm{p} = \frac{1}{\gamma}\mathrm{d}\ln P - \mathrm{d}\ln\rho. \tag{12}$$

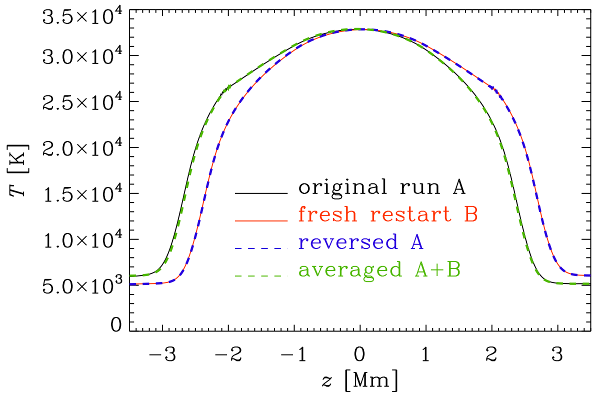

Figure 9: $\Sigma = 2 \times 10^{-12}$, $\mathcal{H} = 5 \times 10^{-5}$, two stable disk states.

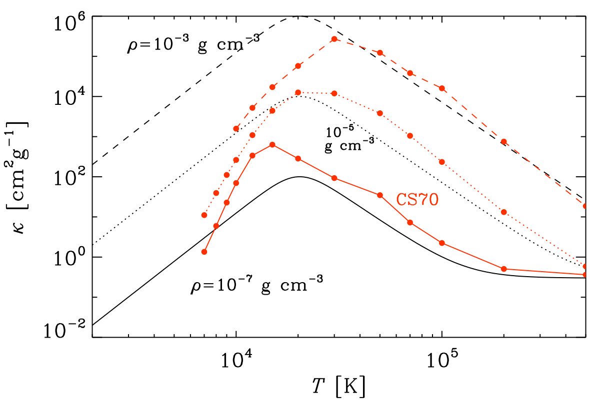

Figure 10: $\kappa_0 = 2 \times 10^{12}\,\mathrm{cm^3\,g^{-1}\,Mm^{-1}}$, $a_{\mathrm{H}}^{-} = 1$, $b_{\mathrm{H}}^{-} = 4$.

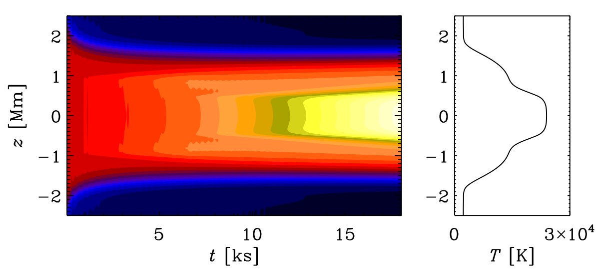

Figure 11: Run with $\kappa_0 = 2 \times 10^{12}\,\mathrm{cm^3\,g^{-1}\,Mm^{-1}}$, $a_{\mathrm{H}}^{-} = 1$, $b_{\mathrm{H}}^{-} = 4$. $H = 2 \times 10^{-5}$, $z_H = 0.5\,\mathrm{Mm}$.

Thus, we have

$$\frac{\mathrm{d}s/c_{\mathrm{P}}}{\mathrm{d}\ln P} = \frac{1}{\gamma} - \frac{1}{\Gamma}, \tag{13}$$

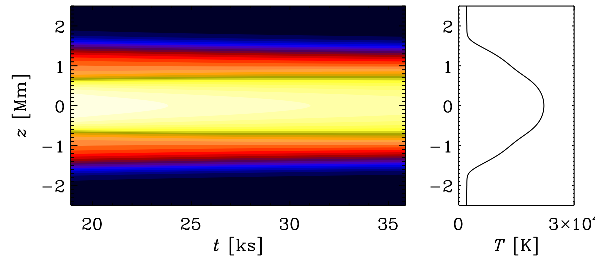

Figure 12: Run with $\kappa_0 = 2 \times 10^{12}\,\mathrm{cm^3\,g^{-1}\,Mm^{-1}}$, $a_{\mathrm{H}}^{-} = 1$, $b_{\mathrm{H}}^{-} = 4$. $H = 0$

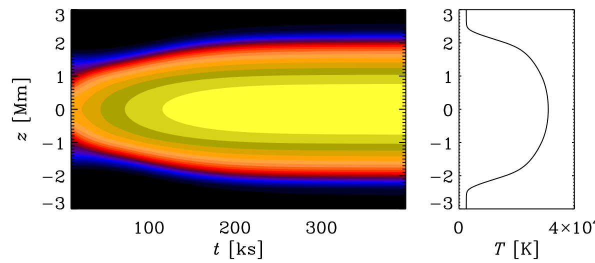

Figure 13: Run with $\kappa_0 = 2 \times 10^{12}\,\mathrm{cm^3\,g^{-1}\,Mm^{-1}}$, $a_{\mathrm{H}}^{-} = 1$, $b_{\mathrm{H}}^{-} = 4$. $\mathcal{H} = 3 \times 10^{-6}$, $z_{\mathcal{H}} = 1\,\mathrm{Mm}$.

which is also known as the superadiabatic gradient, $\nabla - \nabla_{\mathrm{ad}}$. where $\nabla = \mathrm{d}\ln T/\mathrm{d}\ln P$ is the double-logarithmic temperature gradient. Since $\mathrm{d}\ln P/\mathrm{d}z = -H_P$ is a negative quantity (H_P is the pressure scale height), a positive superadiabatic gradient corresponds to a negative specific entropy gradient and thus to a Schwarzschild-unstable stratification. Thus, $\nabla - \nabla_{\mathrm{ad}} > 0$ means $\Gamma > \gamma$.

We have thus three closely related quantities, $n = \mathrm{d}\ln\rho/\mathrm{d}\ln T$, $\Gamma = \mathrm{d}\ln P/\mathrm{d}\ln\rho$, and $\nabla = \mathrm{d}\ln T/\mathrm{d}\ln P$, which are related to each other via

$$\Gamma = 1 + \frac{1}{n} = \frac{1}{1 - \nabla}, \tag{14}$$

$$n = \frac{1}{\Gamma - 1} = \frac{1}{\nabla} - 1, \tag{15}$$

$$\nabla = 1 - \frac{1}{\Gamma} = \frac{1}{n + 1}. \tag{16}$$

For $\gamma = 5/3$, marginal stability corresponds to $n = 3/2 = 1.5$, $\Gamma = \gamma = 5/3$, and $\nabla = 5/2 = 2.5$.

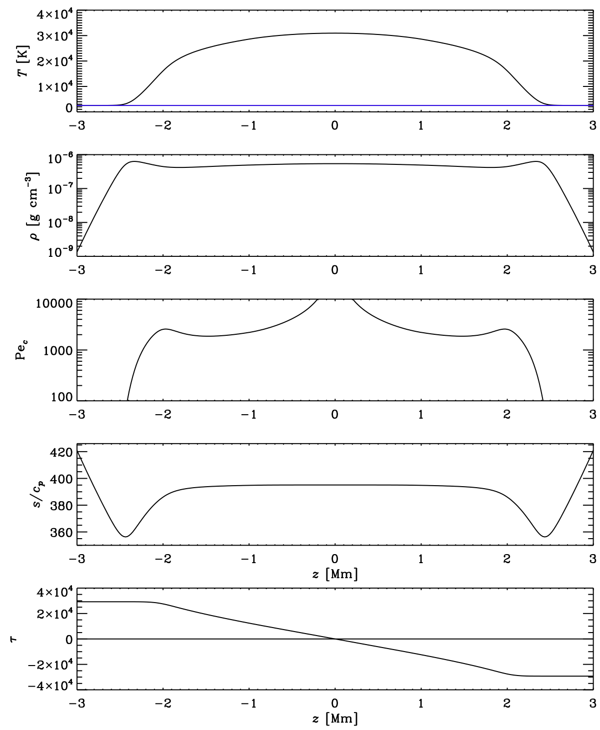

Figure 14: Stratification

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