Notes

December 25, 2011, Revision: 1.40

1 Setup

Show that

$$\operatorname{curl}\sigma_{ab} = -\partial_{\langle b}\omega_{a\rangle} \tag{1}$$

Use definitions:

$$\operatorname{curl} S_{ab} = \epsilon_{cd\langle a}S_{b\rangle d,c} \tag{2}$$

Expand brackets:

$$\operatorname{curl} S_{ab} = \tfrac{1}{2}\epsilon_{cda}S_{bd,c} + \tfrac{1}{2}\epsilon_{cdb}S_{ad,c} - \tfrac{1}{3}\epsilon_{cde}S_{ed,c}\delta_{ab} \tag{3}$$

Insert σ_{ab}

$$\operatorname{curl}\sigma_{ab} = \tfrac{1}{2}\epsilon_{cda}\sigma_{bd,c} + \tfrac{1}{2}\epsilon_{cdb}\sigma_{ad,c} - \tfrac{1}{3}\epsilon_{cde}\sigma_{ed,c}\delta_{ab} \tag{4}$$

or

$$\begin{aligned}\operatorname{curl}\sigma_{ab} = &\left(\tfrac{1}{4}\epsilon_{cda}u_{b,dc} + \tfrac{1}{4}\epsilon_{cda}u_{d,bc} - \tfrac{1}{6}\epsilon_{cda}u_{e,ec}\delta_{bd}\right)\\ &+\left(\tfrac{1}{4}\epsilon_{cdb}u_{a,dc} + \tfrac{1}{4}\epsilon_{cdb}u_{d,ac} - \tfrac{1}{6}\epsilon_{cdb}u_{e,ec}\delta_{ad}\right)\\ &-\tfrac{1}{6}\epsilon_{cde}u_{e,dc}\delta_{ab} - \tfrac{1}{6}\epsilon_{cde}u_{d,ec}\delta_{ab} + \tfrac{1}{9}\epsilon_{cde}u_{f,fc}\delta_{ab}\delta_{de}\end{aligned} \tag{5}$$

First terms vanish

$$\operatorname{curl}\sigma_{ab} = \left(\tfrac{1}{4}\epsilon_{cda}u_{d,bc} - \tfrac{1}{6}\epsilon_{cba}u_{e,ec}\right) + \left(\tfrac{1}{4}\epsilon_{cdb}u_{d,ac} - \tfrac{1}{6}\epsilon_{cab}u_{e,ec}\right) \tag{6}$$

The last terms also vanish, i.e.,

$$\operatorname{curl}\sigma_{ab} = \tfrac{1}{4}\epsilon_{cda}u_{d,bc} + \tfrac{1}{4}\epsilon_{cdb}u_{d,ac} \tag{7}$$

Part II, consider next right-hand side:

$$-\partial_{\langle b}\omega_{a\rangle} = -\tfrac{1}{2}\partial_b\omega_a - \tfrac{1}{2}\partial_a\omega_b \tag{8}$$

Insert definition for curl

$$-\partial_{\langle b}\omega_{a\rangle} = -\tfrac{1}{2}\partial_b\epsilon_{acd}u_{d,c} - \tfrac{1}{2}\partial_a\epsilon_{bcd}u_{d,c} \tag{9}$$

Evaluate other derivative

$$-\partial_{\langle b}\omega_{a\rangle} = -\tfrac{1}{2}\epsilon_{acd}u_{d,cb} - \tfrac{1}{2}\epsilon_{bcd}u_{d,ca} \tag{10}$$

Not the same (by sign and factor 1/2)!?

$Header: /var/cvs/brandenb/tex/etc/notes.tex,v 1.40 2011-06-14 22:34:27 brandenb Exp $