1 Introduction

The possibility of small-scale dynamos has been studied since the early work of B50, who assumed that the statistical properties of magnetic fields agree with those of vorticity. The validity of this assumption was questioned by Mes65, who identified the need for an unexplained magnetic energy supply supply at large length scales.

The topic of small-scale dynamos moved somewhat into the background with the discovery of large-scale dynamos driven by the helicity or α effect (SKR66,Mof78,KR80). With the advent of direct numerical simulation (DNS) of turbulence, the study of small-scale dynamos was picked up again by Mene81 and Kida91, although the work of Kaz68 became routinely quoted only since the 2000s.

The original theory of Kazantsev68 was linear, so it only described the early kinematic growth phase of the dynamo. Furthermore, it assumed that the velocity was smooth and of large scale only. In the framework of turbulence, this could be realized if the magnetic Prandtl number $\mathrm{Pr_M} \equiv \nu/\eta$ is large, i.e., if the viscosity ν is much larger than the magnetic diffusivity η, making therefore the velocity field then much smoother than the magnetic field.

Kazantsev's theory yielded as a solution for the eigenfunction of the magnetic energy spectrum E_M proportional to the McDonald function, which scales with wavenumber k as $k \propto k^{3/2}$. Such scaling was indeed confirmed in a number of different DNS (). It is important to recall, however, that this is only expected for large values of $\mathrm{Pr_M}$. In the opposite limit of $\mathrm{Pr_M} \ll 1$, the spectral slope may be smaller. **?** confirmed a $k^{7/6}$ scaling for $\mathrm{Pr_M} = 0.1$, as was expected based on an earlier theory of Subramanian.

In the meantime, there has been significant work on decaying turbulence. Much of this was motivated by applications to the early universe (**???**). An important question here is how rapidly the magnetic energy decays and how rapidly the correlation length of the turbulence increases. It has been argued that this may depend on the slope of the subinertial range spectrum, i.e., on the exponent α in the magnetic energy spectrum $|EM(k) \propto k^\alpha$ (**?**). **?** found the possibility of inverse cascading, i.e., an increase of the spectral power for small k and a rapid decrease of ξ_M when α is large enough.

In the simulations of **?**, an initial k^4 spectrum was assumed. The value $\alpha = 4$ was later argued to be a general consequence of the requirement of causality, i.e., the requirement that the magnetic field is uncorrelated over different positions, and the fact that $\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0$ **?**. Such a k^4 spectrum is usually referred to as a Batchelor spectrum. Subsequent simulation showed that, in the presence of magnetic helicity, a k^4 spectrum develops automatically, even when the initial spectrum was shallower, e.g., $\propto k^2$, which is called a Saffman spectrum in hydrodynamic turbulence without helicity (**?**). However, subsequent work showed that this is only true because of the presence of magnetic helicity and that non-helical turbulence with an initial Saffman spectrum preserves its slope (**?**).

Many of the MHD decay studies where done for magnetically dominated turbulence, i.e., the initial magnetic energy density is large compared with the kinetic energy density of the turbulence. This precludes the investigation of dynamo action, i.e., the conversion of kinetic energy into magnetic. Simulations of **?** showed that a nearly exponential increases of magnetic energy is still possible when the initial magnetic energy density is small enough.

To summarize, in decaying MHD turbulence, the magnetic energy spectrum can have a k^2 or a k^4 spectrum, depending on the initial conditions. For nonhelical turbulence, in particular, there is no reason to expect a k^4 spectrum, unless the causality argument of **?** is invoked. Earlier work did show a k^4 spectrum of the magnetic field in the kinetically dominated case; see Figure 8 of **?**, but this was in the presence of helicity. Moreover, there was no indication of a $k^{3/2}$ Kazantsev spectrum. This could perhaps be related to the fact that in those simulations, the magnetic Prandtl number was chosen to be unity, i.e., not $\gg 1$. There remained therefore the question, how the Batchelor k^4 spectrum, the Saffman k^2 spectrum, and the Kazantsev $k^{3/2}$ spectrum are related to each other. This is the topic of this paper, where we consider forced turbulence with a very weak initial seed magnetic field. We consider DNS with a resolution of $N^3 = 1024^3$ mesh points, which is still not large enough to cover all turbulent subranges in one simulation, but it is still low enough to be able to experiment with different cases. We therefore compare simulations with different values of $\mathrm{Pr_M}$ and k_f. It will turn out that all three spectra, the Batchelor, Saffman, and Kazantsev spectra are being reproduced in the small-scale dynamo problem if the magnetic Prandtl number is large enough; the the Batchelor and Saffman spectra are being found in the subinertial range during the kinematic and saturated growth phases, respectively, and the Kazantsev $k^{3/2}$ spectrum is found in the magnetic inertial range during the kinematic phase only.

2 Basic equations

We consider weekly compressible isomer turbulence with and I Sosamann equation of state come out where the pressure of P is proportional to the density of raw with P raw and CS being the Jesus Amazon speed. We solve the magnetic for the Mukhtar McVie search for the magnetic actor potential A, so it be cloudy. The full set of results for the evolution of E, the first set of equations for the evolution of a, the velocity you, and the logarithmic density Ellen rule are thats given by,

Where rule is a F is a magnetic is an unheated girl forcing function consisting of plain waves proportion of the tour OK come over OK changes at each time step, making the forcing function Della correlated in time. We select K I randomly from a find that set of actress who is components multiple teachers of K-1 to come out there and is the side length of our Cartesian domain of volume at three.

We always present time averaged spectra, and E, which is straightforward for the kinetic energy and the magnetic energy in the saturated regime, but in the kinematic phase, EM is exponentially growing, so we average the compensated spectra?

3 Results

To begin with, we consider a case with $\mathrm{Pr_M} = 1$ and $k_\mathrm{f} = 30$. We show averaged kinetic and magnetic energy spectra for the kinematic and saturated phase of the dynamo.

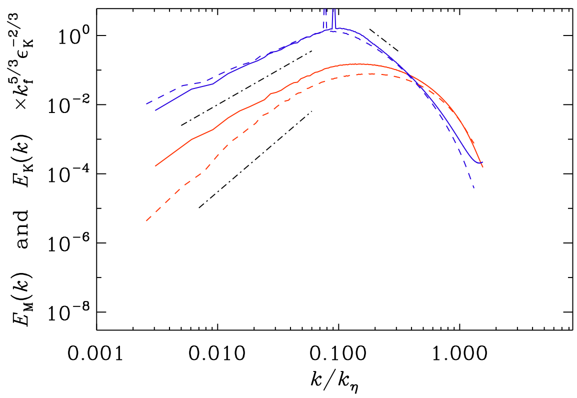

Figure 1: `pspec_comp_kinsat`

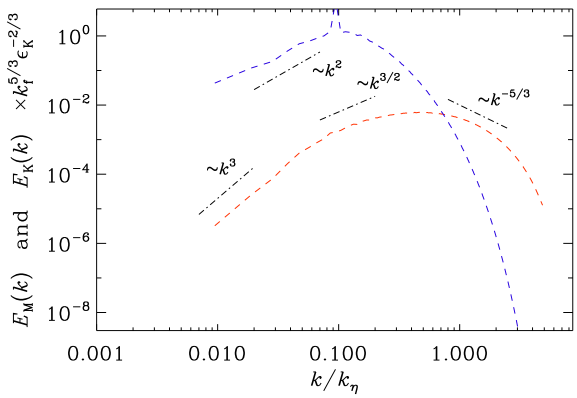

Figure 2: `pspec_comp_kinsat_Pm10`

Pm=100
Pm=50

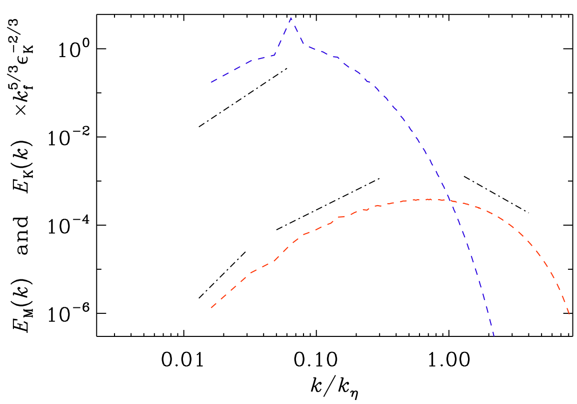

Figure 3: `pspec_comp_kinsat_Pm30`

Pm=100
Pm=50

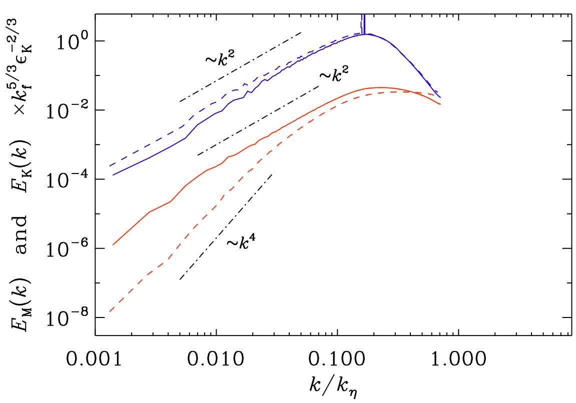

Figure 4: `pspec_comp_kinsat_Pm1`