The timing properties of the pulsar, PSR1509−58, greatly strengthen the identification of MSH15−52 with the supernova of AD 185. It is the second-youngest known radio pulsar after the Crab pulsar. Given the measured pulsar frequency ν, frequency derivative $\dot{\nu}$ and braking index n, we can calculate the characteristic age (in 1982) of the pulsar, $\tau = -\nu/(n-1)\dot{\nu} =$ 1.69 kyr (ref. 19). The true age may be slightly greater if the braking index increases with time, or less if the initial spin frequency was not much greater than the current frequency. Without an assumption of a variable braking index, the characteristic age of the pulsar excludes the 1.8-kyr-old supernova at about the 2σ level[19], but this is almost certainly due to a slight corruption of the braking index by red timing noise such as that observed in nearly all young pulsars[20]. In any case, it is hard to imagine that the close match between the pulsar age and that of SN185 can be coincidental. Even with the most generous estimates of the galactic supernova rate, the chances of two different supernovae in the direction of α and β Cen within a few hundred years are remote.

Despite the evident youth of PSR1509−58, its remnant is sometimes assigned an age of ~10 kyr, on the basis of a model of Sedov expansion into a moderately dense (1 cm^{-3}) medium[14]. Measurements of the optical filaments in the bright knot RCW86 also indicate that neither free nor adiabatic expansion is consistent with their slow velocities[21]. There is no problem reconciling the remnant and pulsar ages, however, if the remnant's initial expansion was into an underdense medium (~0.01 cm^{-3}) such as might be expected in the stellar wind bubble of the massive progenitor star[14,22,23]. Interaction with a high-density medium on the northwest side (towards the galactic plane) would account for both the enhanced luminosity and the slow expansion of the knots.

It is straightforward to make a rough estimate of how bright the supernova would have been in China. If we assume an absolute magnitude at supernova peak of about −19, a distance of 4.2 kpc and an absorption $A_V \approx 3$ (absorption varies across the remnant; this is a typical value[24]), the apparent magnitude at peak would be about −3, similar to a bright planet. With atmospheric extinction, due to its low elevation, the apparent magnitude from Lo-Yang would be about −1.

On 7 December 185, MSH15−52 was only about a week past its heliacal rising, and was at an elevation of ~2.9° at sunrise, high enough for an object of magnitude $m = -1$ to be observed as a morning star. Note that there is no indication in the record that SN185 was ever observable during daylight hours. The coincidence of the reported date of appearance of the guest star and its heliacal rise date suggests that the supernova probably occurred during the weeks (or months) before 7 December, but was unobservable until then. Low elevation (and possibly weather) may account for the week's delay of its discovery after heliacal rising.

On 5 July 186, MSH15−52 was still at an elevation of ~4.2° at sunset, and was above the horizon for 1.5 hours after sunset, so it would have been easy to observe as an evening star. By the end of the month, however, the source set before the Sun, hence its disappearance. No error in the record need be assumed. By the second heliacal rise date, a year after the initial discovery, the guest star would have faded below naked-eye visibility.

Although the good agreement of the heliacal rising and setting dates of MSH15−52, together with the age of the pulsar PSR1509−58, form a compelling case for the identification of MSH15−52 with SN185, this identification is not conclusive. Several observations are desired. The first are high-resolution radio maps of both MSH15−52 and MSH14−63 at several epochs, to measure the proper expansion of both remnants. In both cases, an age of 1,800 years suggests an expansion rate of ~1 arcsec per year, or a few times smaller if the remnant has recently gone from free to adiabatic expansion. It is important to measure the expansion of the low-luminosity radio shells, rather than the higher-luminosity filaments which may arise in high-density clouds. Second is a measurement of the distance to MSH14−63, by H I absorption if possible. A firm distance limit of more than ~2 kpc will rule it out as a candidate.

The identification of MSH15−52 with SN185 is interesting history, but also important astronomy. MSH15−52 is a thousand years older than the next-oldest supernova for which a secure age exists; its study will provide insight into the evolution of remnants in early middle-age. Published studies of PSR1509−58 include less than 2 years of timing data[19]; with a decade now since its discovery it should be possible to refine the $\ddot{\nu}$ measurement of PSR1509−58, and hence its 'timing age'. This will have implications for the early time evolution of pulsar braking indices, or, possibly, the initial spin period of the neutron star. □

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Numerical calculations of fast dynamos in smooth velocity fields with realistic diffusion

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MANY astrophysical magnetic fields are thought to arise by dynamo action due to internal fluid motions, but the natural timescale for magnetic field growth is the diffusion timescale, which in realistic astrophysical applications is very large[1]. A fast dynamo is one that operates on the much shorter turnover timescale of the generating fluid flow, and the analytical intractability of smooth flows with diffusion has prompted the use of many ingenious models[2–10], differing from the true problem in having a modified or time-dependent diffusion or singularities in the flow field. Here we adopt a straightforward approach and present numerical computations of linear kinematic dynamos associated with periodic smooth flows, with diffusion explicitly included. Examples of time-varying flows depending on two spatial coordinates give convincing evidence of fast dynamo action for diffusion times up to 10,000 times greater than the turnover time. A three-dimensional steady flow shows similar behaviour, although computations have not been

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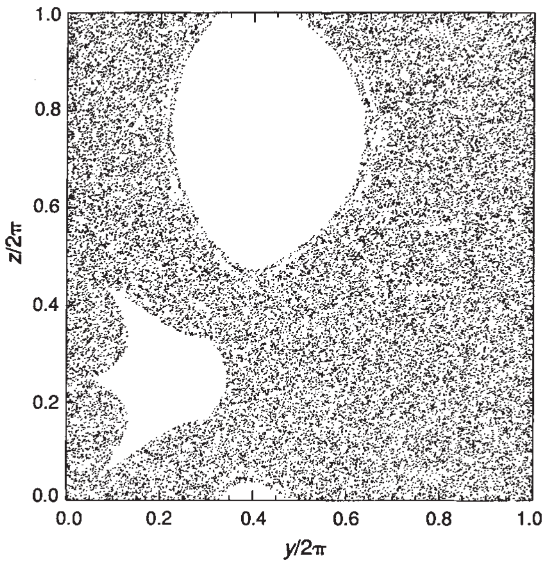

FIG. 1 Poincaré section for the CP flow at $t=0$ in the (y–z) plane (the x-component of **u** is suppressed). Points are plotted at equal time intervals $2\pi/\omega$, and y and z values are plotted modulo 2π.

carried out so far and the asymptotic behaviour is less clear. All these flows have large regions where particle paths are chaotic.

As a simple model of the dynamo process, we prescribe a velocity field **u** and seek exponentially growing solutions to the induction equation for the magnetic field **B**

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{R_m} \nabla^2 \mathbf{B}$$

This is known as the kinematic dynamo problem[11,12]. The equation has been scaled with a characteristic length L, velocity U and time L/U. The quantity $R_m = UL\mu_0\sigma$ is a nondimensional measure of the fluid conductivity σ, and is enormous in astrophysics (typically of order 10^{10}). With this scaling, the flow turnover time is of order 1 and the electromagnetic diffusion time of order R_m.

Although many flows are known that give a growing **B** (refs 11, 12), nearly all of these have the feature that the growth rate tends to zero as $R_m \to \infty$; such flows have been termed slow dynamos[1]. These are supposedly ineffective in astrophysics: the resulting fields would have grown insufficiently during the age of the Universe[1], and short-term variations such as the solar cycle are hard to explain with slow dynamos. Accordingly, attention has now focused on the possibility of fast dynamos, where the growth rate becomes independent of R_m as $R_m \to \infty$. Because analysis has proved exceptionally difficult in this (singular) limit, various adjustments have been made to the problem to help in detecting the asymptotic behaviour. Singularities have been allowed in the velocity[4] and the vorticity[5]; in these cases the speed of the resulting dynamos is apparently due to the singularities themselves. Velocity fields that are discontinuous in time[2] and intermittent diffusion rates[2,3] have also been used; these are less controversial, although a formal proof that the solutions merge with those of the induction equation in the large R_m limit remains elusive. Other work has focused on solutions of the diffusionless equations[7,8]; again, there is no formal connection with the finite-diffusivity case. A more recent approach[9] uses a stochastic diffusion model, integrating along the particle paths (for a review, see ref. 10). A numerical demonstration of fast dynamo action for two pulsed Beltrami vortices was reported by Otani[14], but no details have subsequently been published. Thus there still seems to be scope for high-accuracy numerical experiments with judiciously chosen velocity fields.

Motions that are chaotic are prime candidates for fast dynamos. This is because in the perfectly conducting case, magnetic field lines are advected like fluid line elements[11,13]. Chaotic flows stretch these exponentially, providing a powerful mechanism for growth of the field. Indeed, flows with no chaos and no hyperbolic stagnation points always yield slow dynamos[20]. The 'ABC' flows[15]

$$\mathbf{u} = (A \sin z + C \cos y,\ B \sin x + A \cos z,\ C \sin y + B \cos x)$$

where A, B and C are constants, are known to possess chaotic regions when $ABC \neq 0$ (refs 16, 17), and their dynamos have been examined to see whether they are fast, with inconclusive results[18,19]. Research has mainly concentrated on the case $A = B = C$, although unfortunately this special case has very small chaotic regions.

Chaotic flows and their induced fields are by their nature highly resistant to analytical methods. On the other hand, numerical methods cannot currently resolve values of R_m higher than a few hundred in three space dimensions[19] and this is not really high enough. It is known, however, that flows depending on only two space dimensions can be chaotic if they are time-dependent. Their dynamos can be investigated with a two-dimensional code, thus enabling much higher values of R_m to be attained. Accordingly, let us consider modifying the integrable ABC flow with $B = 0$ so that its phasing depends on time in two possible ways:

$$\mathbf{u} = (A \sin(z + \sin \omega t) + C \cos(y + \cos \omega t),\ A \cos(z + \sin \omega t),\ C \sin(y + \cos \omega t))$$

$$\mathbf{u} = (A \sin(z + \cos \omega t) + C \cos(y + \cos \omega t),\ A \cos(z + \cos \omega t),\ C \sin(y + \cos \omega t))$$

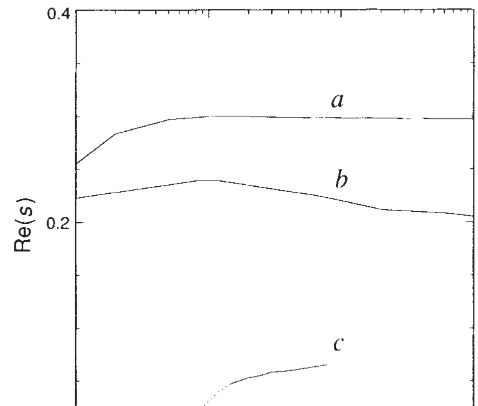

FIG. 2 Real part of the growth rate S plotted against R_m (log scale) for the three flows discussed in the text. *a*, CP flow, $k=0.57$. *b*, LP flow, $k=0.62$. *c*, three-dimensional flow. The dotted portion of the latter is determined with lower accuracy.

These are now non-integrable and possess large chaotic regions (see Fig. 1). The first is 'circularly polarized' (CP) and stirs the basic $A(B)C$ flow around, whereas the second is 'linearly polarized' (LP), and shakes it back and forth parallel to the $y = z$ diagonal. In what follows we treat only the case $A = C = \sqrt{3/2}$, $\omega = 1$.

The induction equation is solved numerically by writing the magnetic field as a triple Fourier series and timestepping the Fourier amplitudes; the method is a straightforward adaptation of that in ref. 19. Because $B = 0$, each mode proportional to e^{ikx} evolves independently of modes with different k. Thus we can fix k and compute the resulting $\mathbf{B}$ with a two-dimensional code. The y-z periodicity of the solution is taken for convenience to be the same as that of the flow. As k is a free parameter, one has to optimize over k to find the value k_c giving the eigenvalue with largest real part. The solution with zero k is known to decay from the two-dimensional analogue of Cowling's theorem[1]; for large enough k we should find decay, although k_c may grow with R_m. This is seen in Soward's[5] infinite-vorticity fast dynamo, which uses a very similar but steady velocity field, and for which $k_c \approx R_m^{1/2}$. Surprisingly, k_c is here of order 1 and varies little if at all with R_m, being 0.57 (CP flow) and 0.62 (LP flow): the behaviour of the real part of the growth rate as a function of R_m for the two flows is shown in Fig. 2. The CP flow in particular provides convincing evidence for fast dynamo action, the real part scarcely changing between $R_m = 50$ and $R_m = 10{,}000$. The LP flow is only marginally less convincing. The real parts of the growth rates for the magnetic energy tend to 0.3 and 0.2 respectively; there are only very small fluctuations in the energy due to the temporal variation of the velocity field. For comparison, the largest Lyapunov exponents in the chaotic regions are 1.43 and 1.17. All Fourier amplitudes are initially filled equally, and as time t advances, there is a period where the energy falls as the high-order modes decay. For large enough R_m, the energy levels off at $t \approx 10$ and exponential growth begins at $t \approx 25$. Thus the form of the eigenfunction seems to be established on the turnover timescale. This was confirmed by running the case $R_m = 400$ for a whole diffusion time. The code itself was checked against a multiple-timescale analysis at low R_m.

The structure of the eigenfunction for the CP flow is shown in Fig. 3 (the LP solution looks rather similar but without the structures parallel to $y = z$). A detailed comparison with the velocity field is difficult because of the three-dimensionality, but broadly speaking the field structures line up along the unstable directions of stagnation points of the y-z flow. The thickness of the structures goes down as R_m increases, presumably scaling[21] as $R_m^{-1/2}$, the behaviour found in analogous, analytically tractable situations[22,23] (although here the structure may be fractally pleated in the limit of infinite R_m). As k is of order 1, the structures are like curved sheets. As $R_m \to \infty$ these eigenfunctions must tend to generalized functions although, remarkably, their growth rate tends to a finite limit.

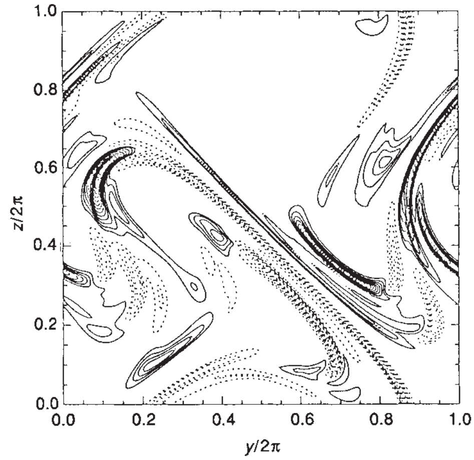

FIG. 3 Contours of B_x at $x = 0$: CP flow, $k = 0.57$, $R_m = 2{,}000$.

Finally we give results for the three-dimensional steady flow $\mathbf{u} = (\sin z, \sin x, \sin y)$, which is the $A = B = C = 1$ flow with the cosines omitted, making it roughly twice as efficient to compute (but still requiring a three-dimensional code). Poincaré sections reveal that almost all the flow region is chaotic, but in contrast to the ABC flow, the helicity density $\mathbf{u} \cdot \nabla \times \mathbf{u}$ has zero spatial average. This is supposedly unconducive to dynamo action but, as shown in Fig. 2, the flow operates perfectly satisfactorily as a dynamo, and in fact seems likely to be fast. The imaginary part of the growth rate is zero. As yet we have only achieved results for R_m up to 800 (using $(96)^3$ resolution), with no information on eigenfunction structure. Further computations are planned, and we should be able to obtain resolved solutions for values of R_m up to 2,000.

These computations provide strong evidence for the existence of fast dynamos, but no proof. For the astrophysicist such a proof is probably academic: all his flows are likely to be highly turbulent and chaotic, and the dynamos that they produce seem likely to be fast. Calculating the fields directly seems possible only by numerical simulation or by the use of theories that parameterize fast dynamo generation (for example, the α-effect[1]). A more practical challenge is to translate fast dynamo ideas and models to more realistic geometries such as spheres and discs surrounded by empty space. Another is to examine the back reaction of the field on the motion. This is interesting because there is a physical inconsistency inherent in kinematic fast dynamos: elementary consideration of the effects of field amplification suggests that however small the seed field, for large R_m the Lorentz force becomes significant after a time much shorter than the diffusion time, albeit over very small volumes. In fact, intermittent fields seem to be common in astrophysics, with strong fields concentrated in small regions; a sunspot is an obvious example. □

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