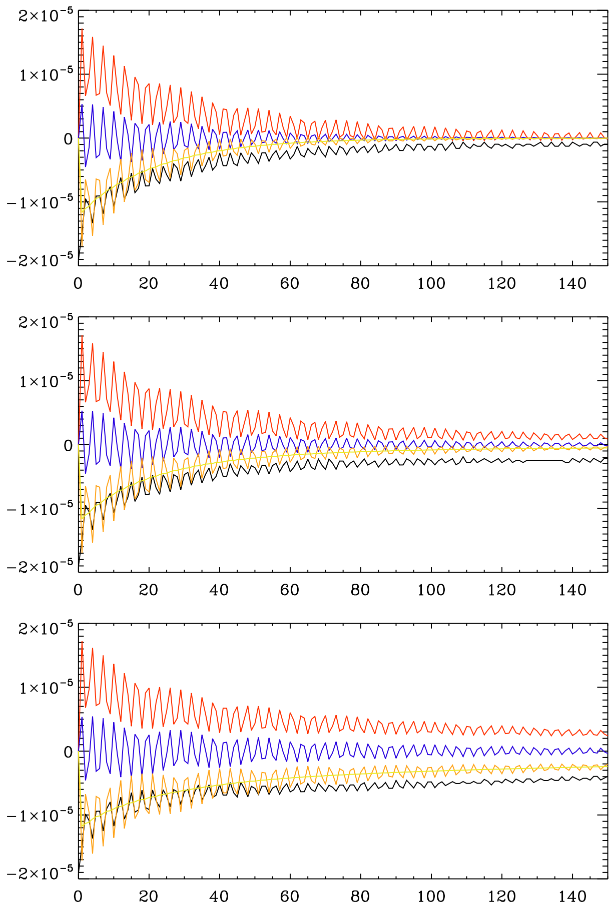

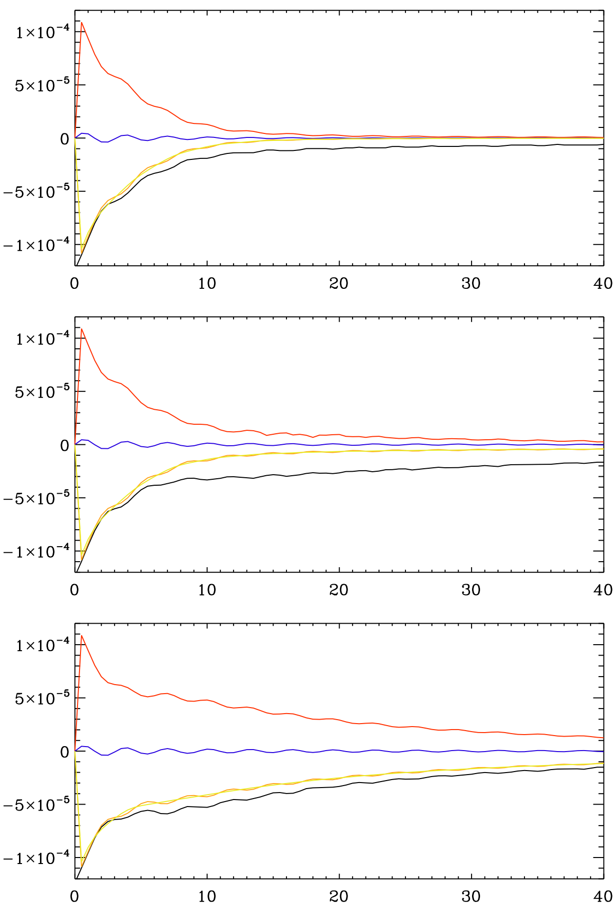

Figure 1: black: $\dot{\mathcal{E}}_{\rm M}$, red: $W_{\rm L}$, blue: $W_{\rm n}$, orange: $W_{\rm i}$, yellow: $W_{\rm n}+W_{\rm i}$, for $\tau_{\rm AD}'=1$ and $\zeta=10^{-9}$ (top), 10^{-5} (middle), and 10^{-3} (bottom), just like in the 2019 paper.

Figure 2: Same as Figure 1, but with 10 times larger γ, so $\tau_{\rm AD}'=10$.

Basic equations

$$\frac{\partial \boldsymbol{A}}{\partial t}=\boldsymbol{u}_{\rm i}\times\boldsymbol{B}-\eta\mu_0\boldsymbol{J}, \tag{1}$$

$$\rho_{\rm i}\frac{{\rm D}\boldsymbol{u}_{\rm i}}{{\rm D}_{\rm i}t}=\boldsymbol{J}\times\boldsymbol{B}-\boldsymbol{\nabla}p_{\rm i}+\boldsymbol{\nabla}\cdot(2\nu\rho_{\rm i}\mathsf{S}_{\rm i})-\rho(\rho_{\rm i}\gamma+\zeta)(\boldsymbol{u}_{\rm i}-\boldsymbol{u}), \tag{2}$$

$$\rho\frac{{\rm D}\boldsymbol{u}}{{\rm D}t}=\rho\boldsymbol{f}-\boldsymbol{\nabla}p+\boldsymbol{\nabla}\cdot(2\nu\rho\mathsf{S})+\rho_{\rm i}(\rho\gamma+\alpha\rho_{\rm i})(\boldsymbol{u}_{\rm i}-\boldsymbol{u}), \tag{3}$$

$$\frac{{\rm D}\ln\rho_{\rm i}}{{\rm D}_{\rm i}t}=-\boldsymbol{\nabla}\cdot\boldsymbol{u}_{\rm i}+\zeta\rho/\rho_{\rm i}-\alpha\rho_{\rm i}, \tag{4}$$

$$\frac{{\rm D}\ln\rho}{{\rm D}t}=-\boldsymbol{\nabla}\cdot\boldsymbol{u}-\zeta+\alpha\rho_{\rm i}^2/\rho, \tag{5}$$

Frictional heating

$$\mathcal{H}_{\rm i}=+\rho(\rho_{\rm i}\gamma+\zeta)(\boldsymbol{u}_{\rm i}-\boldsymbol{u})\cdot\boldsymbol{u}_{\rm i}, \tag{6}$$

$$\mathcal{H}_{\rm n}=-\rho_{\rm i}(\rho\gamma+\alpha\rho_{\rm i})(\boldsymbol{u}_{\rm i}-\boldsymbol{u})\cdot\boldsymbol{u}, \tag{7}$$

So that

$$\mathcal{H}_{\rm i}+\mathcal{H}_{\rm n}=\rho(\rho_{\rm i}\gamma+\zeta)(\boldsymbol{u}_{\rm i}-\boldsymbol{u})^2+\mathcal{R}, \tag{8}$$

where

$$\mathcal{R}=\alpha(\rho_{\rm i}-\rho_{\rm i0})^2-\zeta(\rho-\rho_0)^2 \tag{9}$$

is a residual that vanishes if $\rho_{\rm i}$ and ρ are equal to their iniial values, $\rho_{\rm i0}$ and ρ_0, respectively.

In the 1-fluid approximation, we have, instead, $\epsilon_{\rm AD}=(\tau_{\rm AD}/\rho_0)\langle(\boldsymbol{J}\times\boldsymbol{B})^2\rangle$.

1 Results

$\dot{\mathcal{E}}_{\rm M}<0$, so magnetic energy is lost and goes via $W_{\rm L}$ into kinetic energy of the ions, which is then dissipated via frictional heating.

The oscillations are relaxation oscillations owing to the initial pressure imbalance. Neutral and charged particles oscillate in antiphase.

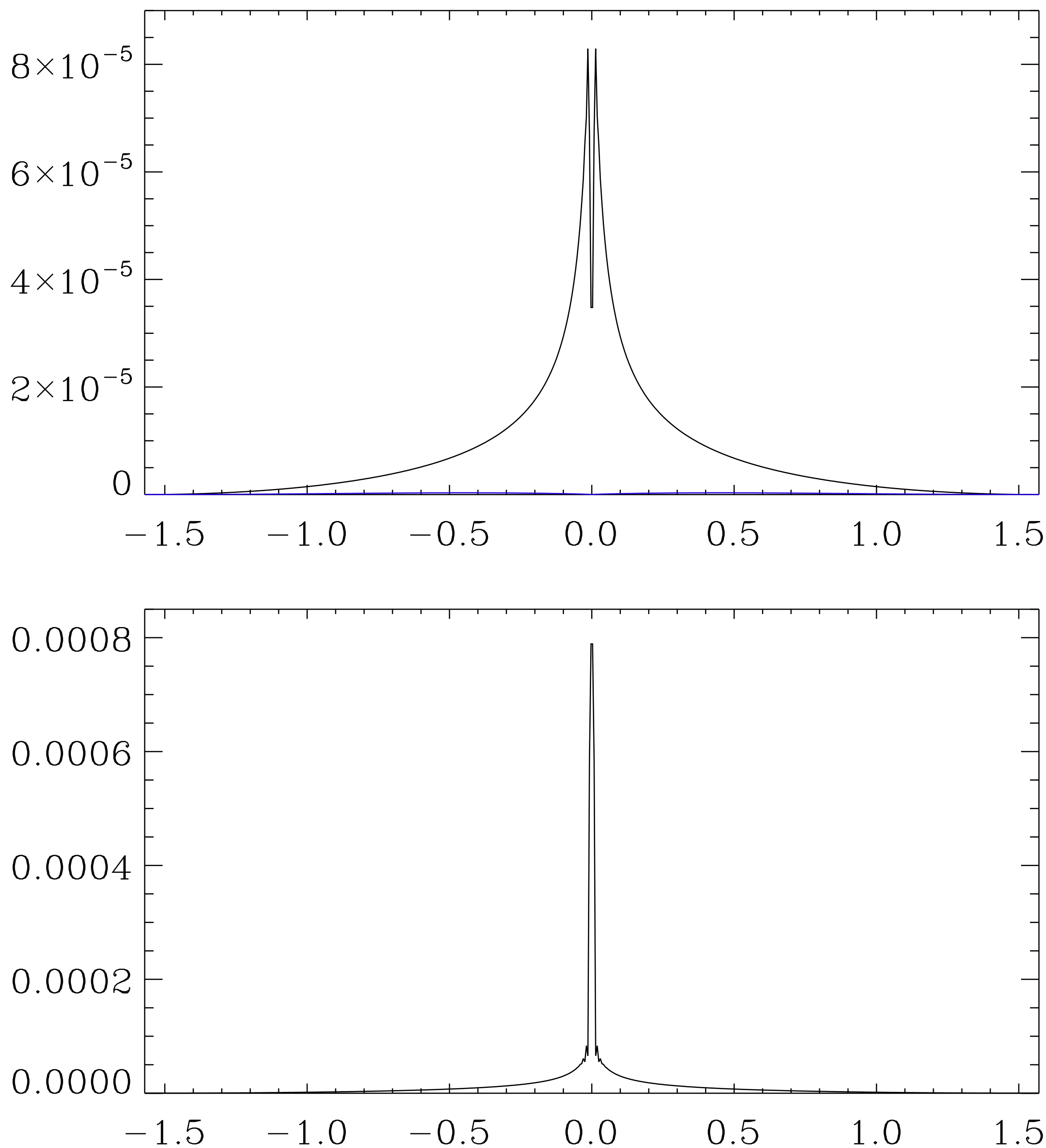

Figure 3: Heating profile of (a) ions (black) and neutrals (blue) in the 2-fluid description, and (b) in the 1-fluid description, for $\tau'_{\rm AD} = 10$, $\zeta = 10^{-3}$, $\alpha = 10^5$, $\gamma = 10^3$, $\rho_{\rm i0} = 10^{-4}$, and $\rho_0 = 1$.