Drag reduction by a turbulent dynamo?

December 30, 2021, Revision: 1.11

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In pipe flows, drag reduction, for example by the addition of polymers, allows the mass flow to be increased while keeping the pressure drop unchanged. To address this problem numerically, we consider a turbulent flow between non-slip boundaries. We begin by considering a situation where the flow is periodic in one of the two cross-stream directions (x) and also periodic in the streamwise direction (y). Boundary conditions are therefore applied only in the z direction at $z = \pm L/2$, which are chosen to be at $z = \pm\pi$. Turbulence is produced by forcing the flow in the volume through a forcing function that consists of random plane waves that can be helical or nonhelical. A magnetic field can emerge solely from dynamo action if the magnetic Reynolds number is large enough. We apply insulating boundary conditions, i.e., the magnetic field on the boundary vanishes, $\boldsymbol{n} \times \boldsymbol{B} = 0$, and there for no electric field into the boundary, $\boldsymbol{n} \cdot \boldsymbol{E} = 0$.

2 The model

We solve the forced isothermal MHD equations for the velocity $\boldsymbol{U}$, the logarithmic density $\ln\rho$, and the magnetic vector potential $\boldsymbol{A}$,

$$\frac{\mathrm{D}\boldsymbol{U}}{\mathrm{D}t} = -c_\mathrm{s}^2 \boldsymbol{\nabla} \ln\rho + \boldsymbol{f} + \boldsymbol{F} + \frac{1}{\rho}\left(\boldsymbol{J} \times \boldsymbol{B} + \boldsymbol{\nabla} \cdot 2\nu\rho\mathsf{S}\right), \tag{1}$$

$$\frac{\mathrm{D}\ln\rho}{\mathrm{D}t} = -\boldsymbol{\nabla} \cdot \boldsymbol{U}, \tag{2}$$

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{U} \times \boldsymbol{B} + \eta\nabla^2\boldsymbol{A}, \tag{3}$$

where $c_\mathrm{s} = \mathrm{const}$ is the sound speed, $\boldsymbol{f}$ and $\boldsymbol{F}$ are forcing functions, $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$ is the magnetic field, $\boldsymbol{J} = \boldsymbol{\nabla} \times \boldsymbol{B}/\mu_0$ is the current density, μ_0 is the vacuum permeability, ν is the viscosity, $\mathsf{S}_{ij} = (\partial_i U_j + \partial_j U_i)/2 + \delta_{ij} \boldsymbol{\nabla} \cdot \boldsymbol{U}$ are the components of the rate of strain tensor S, and η is the magnetic diffusivity. Turbulence is driven by stochastic random waves $\boldsymbol{f}$ that have a different direction at each time step and a mean wavenumber k_f, and the mean flow is driven by the function $\boldsymbol{F} = \hat{\boldsymbol{y}}F$.

In the absence of turbulence, the flow profile is parabolic.

$$U = A\,(\pi - z)(\pi + z) \tag{4}$$

with $\max U = A\pi^2$ and $U'' = 2A$. Use $F = \nu U'' = 2\nu A$. Want $\max U \approx 0.5$, so choose $A = 0.05$, so, for $\nu = 0.01$, we have $F = 2 \times 0.01 \times 0.05 = 10^{-3}$. The mean work done against the laminar viscous force is given by $W_\nu = (\pi F)^2/3\nu$.

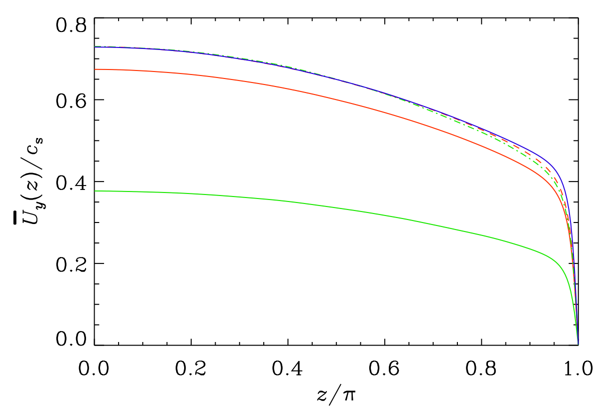

Figure 1: Blue shows the profile for the nonmagnetic run, and red is the a magnetic run with $\mathrm{Pr}_M = 1$ and no helicity. The green line is also a magnetic run, but with $\mathrm{Pr}_M = 2$. The red dashed line is a magnetic run scaled up by a factor 1.082, to show that the profile has also a different shape. Likewise, the dashed green line is scaled up by a factor 1.936. `pcomp_prof`

Table 1 gives a summary of runs. The fluid Reynolds number is in the range 200–300; the smaller values are a result of the suppression of turbulence by the dynamo. The turbulent suppression of the mean flow can be seen by the smallness of the

Table 1:

Run	F	ν	Pr_M	Re	W_F/W_ν	ϵ_T/W_ν	ϵ_M/W_ν	comment
A	5×10^{-4}	2×10^{-4}	1	317	0.075	0.325	—	non-magnetic
B	5×10^{-4}	2×10^{-4}	1	291	0.069	0.311	0.134	small-scale dynamo

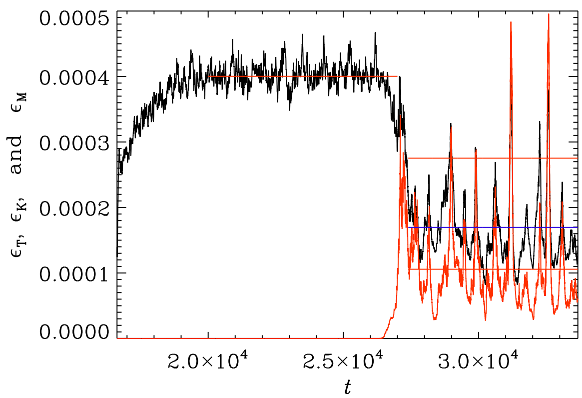

Figure 2: Evolution of ϵ_K for the non-magnetic run (black) and the magnetic run (blue), showing also ϵ_M (red) and $\epsilon_T=\epsilon_K+\epsilon_M$ for this run. Run C with $\mathrm{Pr}_M=2$ (dotted lines) has been restarted from Run B at $t=6000$. `peps_512b_nomag_cont_nof3_nohel`

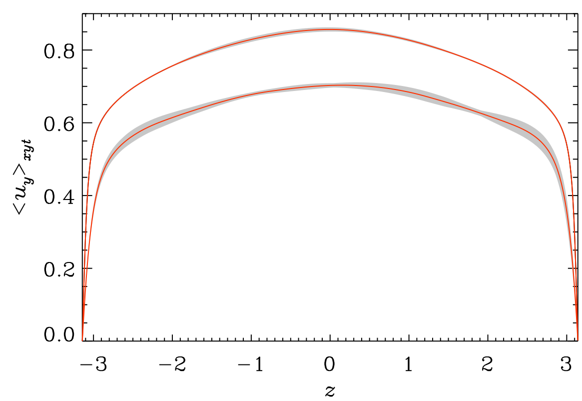

Figure 3: Mean flow profiles for nonmagnetic (upper line) and dynamo-generated magnetic turbulence (lower line).

values of W_F/W_ν. In the absence of turbulence they would be unity, but at the Reynolds numbers considered here, turbulence suppresses the flow speed to between 3% and 8% of the laminar value.

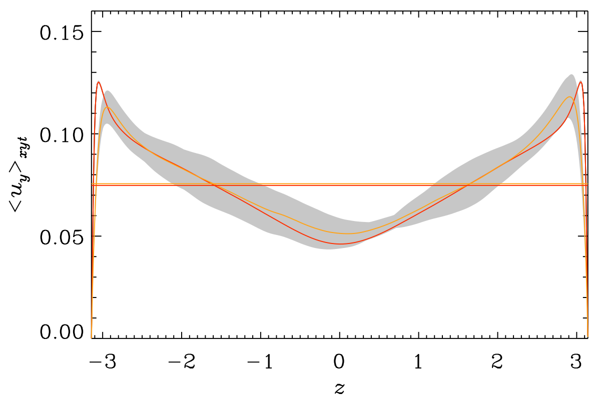

Figure 4: Profile of rms velocity.

References

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Ogilvie, G. I., & Potter, A. T., Magnetorotational-type instability in Couette-Taylor flow of a viscoelastic polymer liquid. *Phys. Rev. Lett.* 2008, **100**, 074503.

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