Table 1
Summary of parameters of our sample.

| $\#$ | $S$ | $B-V$ | $T_{\text {eff }}$ | $\tau_{\text {Noy }}$ | $P_{\text {rot }}$ | $P_{\text {rot }}^{*}$ | $\log \left\langle R_{\text {HK }}^{\prime}\right\rangle$ | Age |
| :--- | ---: | ---: | ---: | ---: | :--- | :--- | :--- | :--- |
| A | 724 | 0.63 | 5856 | 10.8 | 24.2 | 19.7 | -4.79 | 4.3 |
| B | 777 | 0.63 | 5856 | 10.8 | 12.1 | 19.7 | -4.90 | 1.3 |
| C | 801 | 0.68 | 5692 | 13.7 | 24.6 | 21.8 | -4.95 | 3.7 |
| D | 802 | 0.68 | 5692 | 13.7 | 18.4 | 21.8 | -4.95 | 2.2 |
| E | 958 | 0.62 | 5890 | 10.2 | 21.2 | 19.2 | -4.89 | 3.6 |
| F | 965 | 0.72 | 5564 | 15.9 | 25.7 | 23.3 | -4.86 | 3.6 |
| G | 1218 | 0.64 | 5823 | 11.4 | 20.1 | 20.2 | -4.78 | 3.0 |
| H | 1307 | 0.77 | 5408 | 18.2 | 23.5 | 24.8 | -4.95 | 2.7 |
| $\alpha$ | 603 | 0.55 | 6091 | 6.4 | - | 14.7 | -4.74 | $3^{*}$ |
| $\beta$ | 746 | 0.67 | 5725 | 13.1 | - | 21.4 | -4.89 | $3^{*}$ |
| $\gamma$ | 770 | 0.64 | 5823 | 11.4 | - | 20.2 | -4.80 | $3^{*}$ |
| $\delta$ | 785 | 0.66 | 5757 | 12.6 | - | 21.0 | -4.82 | $3^{*}$ |
| $\epsilon$ | 945 | 0.63 | 5856 | 10.8 | - | 19.7 | -4.80 | $3^{*}$ |
| $\zeta$ | 969 | 0.63 | 5856 | 10.8 | - | 19.7 | -5.06 | $3^{*}$ |
| $\eta$ | 991 | 0.64 | 5823 | 11.4 | - | 20.2 | -4.84 | $3^{*}$ |
| $\theta$ | 1004 | 0.72 | 5564 | 15.9 | - | 23.3 | -5.02 | $3^{*}$ |
| $\iota$ | 1048 | 0.65 | 5790 | 12.0 | - | 20.6 | -5.17 | $3^{*}$ |
| $\kappa$ | 1078 | 0.62 | 5890 | 10.2 | - | 19.2 | -4.95 | $3^{*}$ |
| $\lambda$ | 1087 | 0.60 | 5957 | 9.1 | - | 18.2 | -4.90 | $3^{*}$ |
| $\mu$ | 1089 | 0.63 | 5856 | 10.8 | - | 19.7 | -4.97 | $3^{*}$ |
| $\nu$ | 1095 | 0.61 | 5923 | 9.7 | - | 18.7 | -4.73 | $3^{*}$ |
| $\xi$ | 1096 | 0.62 | 5890 | 10.2 | - | 19.2 | -4.86 | $3^{*}$ |
| $o$ | 1106 | 0.65 | 5790 | 12.0 | - | 20.6 | -4.93 | $3^{*}$ |
| $\pi$ | 1212 | 0.73 | 5530 | 16.4 | - | 23.6 | -4.86 | $3^{*}$ |
| $\rho$ | 1248 | 0.58 | 6025 | 8.0 | - | 17.0 | -4.65 | $3^{*}$ |
| $\sigma$ | 1252 | 0.59 | 5988 | 8.5 | - | 17.6 | -4.72 | $3^{*}$ |
| $\tau$ | 1255 | 0.63 | 5856 | 10.8 | - | 19.7 | -4.82 | $3^{*}$ |
| $v$ | 1258 | 0.63 | 5856 | 10.8 | - | 19.7 | -4.90 | $3^{*}$ |
| $\phi$ | 1260 | 0.58 | 6025 | 8.0 | - | 17.0 | -4.78 | $3^{*}$ |
| $\chi$ | 1269 | 0.72 | 5564 | 15.9 | - | 23.3 | -5.02 | $3^{*}$ |
| $\psi$ | 1289 | 0.72 | 5564 | 15.9 | - | 23.3 | -4.88 | $3^{*}$ |
| $\omega$ | 1341 | 0.70 | 5625 | 14.8 | - | 22.6 | -4.79 | $3^{*}$ |
| $\infty$ | 1420 | 0.59 | 5988 | 8.5 | - | 17.6 | -4.79 | $3^{*}$ |
| $\ell$ | 1449 | 0.62 | 5890 | 10.2 | - | 19.2 | -5.13 | $3^{*}$ |
| $[$ | 1477 | 0.68 | 5692 | 13.7 | - | 21.8 | -4.94 | $3^{*}$ |
|  |  |  |  |  |  |  |  |  |

$T_{\text {eff }}$ is in Kelvin, $\tau_{\text {Noy }}$ and $P_{\text {rot }}$ is in days, and age is in Gyr. $P_{\text {rot }}^{*}$ (in days) is computed from Equation (1) assuming an age of $t=4 \mathrm{Gyr}$,

Many of the rotation periods reported in the earlier work of BG18, have been revised. We are now left with only eight single stars with reliable periods in our sample. Five of them are from the original sample, while for another three stars, no periods were previously determined (S724, S777, and S802). Those stars are listed in Table 1 with uppercase letters. The number of stars with known values of $\left\langle R_{\mathrm{HK}}^{\prime}\right\rangle$, but without measured periods, is now 27. Those are denoted by lowercase Greek letters. Contrary to our earlier results, the average gyrochronological age is now closer to 3 Gyr than to 4 Gyr . The computed rotation periods (Mamajek \& Hillenbrand 2008),

$$
\begin{equation*}
P_{\mathrm{rot}}^{*}=0.407(B-V-0.495)^{0.325} t^{0.565} \tag{1}
\end{equation*}
$$

where the asterisk is used to distinguish the computed value from the measured one, are therefore based on an age of $t=$ 3 Gyr , and are also listed in Table 1.

In Figure 1 we show a revised comparison of our results with those of BMM in a rotation-activity diagram of $\log \left\langle R_{\text {HK }}^{\prime}\right\rangle$ versus $\tau_{\text {Noy }} / P_{\text {rot }}^{*}$ using the semi-empirical turnover times for all stars with measured rotation periods and turnover times of Noyes et al. (1984),

$$
\begin{equation*}
\log \tau_{\text {Noy }}=1.362-0.166 x+0.03 x^{2}-5.3 x^{3} \tag{2}
\end{equation*}
$$

with $x=1-(B-V)$ and for $B-V<1$. As in BG18, the gray boxes show error bars in $\left\langle R_{\mathrm{HK}}^{\prime}\right\rangle$ and $P_{\text {rot }}$. The stars of BMM are described by the fit $\log \left\langle R_{\mathrm{HK}}^{\prime}\right\rangle \approx$ $\log \left(\tau_{\text {Noy }} / P_{\text {rot }}\right)+\log c$, where $\log c \approx-4.63$. As seen before, the activity tends to increase for the stars of M67 as $\tau_{\text {Noy }} / P_{\text {rot }}$ decreases. The fit computed from the residual between $\log \left\langle R_{\mathrm{HK}}^{\prime}\right\rangle$ and $\log \left(\tau_{\text {Noy }} / P_{\text {rot }}\right)$,

$$
\begin{equation*}
\log \left\langle R_{\mathrm{HK}}^{\prime}\right\rangle-\log \left(\tau_{\text {Noy }} / P_{\mathrm{rot}}\right)=\log c_{1}+\rho \log \left\langle R_{\mathrm{HK}}^{\prime}\right\rangle \tag{3}
\end{equation*}
$$

is denoted by $\log c$ in the inset of Figure 1, where $c$ is a function of $\left\langle R_{\mathrm{HK}}^{\prime}\right\rangle$. The parameters in Equation (3) are $\log c_{1} \approx$ 4.28 and $\rho \approx 1.84$. This fit is shown in the upper inset of Figure 1 as a solid line. Solving for $\log \left\langle R_{\mathrm{HK}}^{\prime}\right\rangle$ gives

$$
\begin{equation*}
\log \left\langle R_{\mathrm{HK}}^{\prime}\right\rangle=\log c_{2}+\mu_{2} \log \left(\tau_{\mathrm{Noy}} / P_{\mathrm{rot}}\right) \tag{4}
\end{equation*}
$$

where $\log c_{2}=\mu_{2} \log c_{1} \approx-5.13$ with $\mu_{2}=(1-\rho)^{-1} \approx$ -1.20 . The direct fit for the eight stars gives $\log c_{2}^{*} \approx-4.96$ and $\mu_{2}^{*}=-0.36$; see the dashed line in Figure 1.

In Figure 2, we show a rotation activity diagram using $\tau_{\text {Noy }} / P_{\text {rot }}^{*}$ based on gyrochronology; see Equation (1). Here the uprise of activity at slow rotation is clearly more pronounced. BG18 interpreted this increase of activity at large Rossby numbers as analogous to what was found by Karak et al. (2015). In their case, an uprise of activity occurred in the regime of antisolar differential rotation, where the absolute differential rotation was found to be larger than in the regime of solar-like differential rotation. This interpretation has now been challenged based on newer simulations that no longer show a significant magnetic energy increase with decreasing differential rotation (Viviani et al. 2019).

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Figure 1. $\log \left\langle R_{\mathrm{HK}}^{\prime}\right\rangle$ versus $\log \left(\tau_{\mathrm{Noy}} / P_{\mathrm{rot}}\right)$ for the stars of M67 with known rotation periods as green uppercase letters, the F and G dwarfs of BMM as blue italics characters, the K dwarfs of BMM as red roman characters, and the four stars of SB with $P_{\text {rot }} / \tau_{\text {Noy }} \geq 2.4$ as orange numbers 1-4. On the upper abscissa, the Rossby number $P_{\text {rot }} / \tau_{\text {Noy }}$ is given. The dashed-dotted line shows the fit of BMM, whereas the solid line represents a fit to the residuals in Equation (4) for the nine stars with $\log \left\langle R_{\mathrm{HK}}^{\prime}\right\rangle \geq-4.85$. The dashed line is a direct fit to the eight stars and the dotted line shows Equation (4). The arrow indicates the anticipated evolution with increasing age $t$. Some of the symbols have been shifted slightly to avoid overlapping. The Sun corresponds to the blue italics $a$. The upper inset shows the residual $\log c$ versus $\log \left\langle R_{\mathrm{HK}}^{\prime}\right\rangle$ for the stars of M67 as green filled circles, the F and G dwarfs of BMM as blue diamonds, and the K dwarfs of BMM as red crosses.


Figure 2. Similar to Figure 1, but now with rotation periods computed from $B-V$ using Equation (1) and the assumption that M67 is 4 Gyr old. (The green symbols would end up further to the left if we assumed instead an age of 5 Gyr.) Here all stars are included—not just those for which $P_{\text {rot }}$ would also be available; see Table 1.

