Scalings

March 18, 2020, Revision: 1.1

1 Solving the disk structure by integration

In our approach, we prescribe a heating function, $\mathcal{H}(z)$, in the energy equation

$$\frac{\mathrm{d}F}{\mathrm{d}z} = \mathcal{H}(z) \tag{1}$$

which leads to a fixed flux profile, $F(z) = F_\infty f(z)$, where $f(z)$ goes from 0 (in the midplane) to 1 for $z \geq L$. In our approach, the flux at infinity is given in terms of $\dot{M}$ via

$$F_\infty = \mathcal{H}_0 L = \frac{\dot{\Sigma}}{3\pi}(\tfrac{3}{2}\Omega)^2 = \sigma_{\mathrm{SB}} T_{\mathrm{eff}}^4, \tag{2}$$

which thus also determines the effective temperature. Next, we have

$$\frac{\mathrm{d}T}{\mathrm{d}z} = -F(z)/K(\rho, T) \tag{3}$$

$$\frac{\mathrm{d}P}{\mathrm{d}z} = -\rho g \tag{4}$$

where $g = \Omega^2 z$ is the local gravitational acceleration. Divide by each other, and use $P = \mathcal{R}T\rho/\mu$, so

$$\nabla_{\mathrm{rad}} \equiv \frac{\mathrm{d}\ln T}{\mathrm{d}\ln P} = \frac{\mathcal{R}}{\mu}\,\frac{F_\infty}{K\Omega^2}\,g(z) \tag{5}$$

where

$$g(z) = f(z)/z = \begin{cases} 1 & (\text{for } z \leq L), \\ 1/z & (\text{for } z > L). \end{cases} \tag{6}$$

and

$$\dot{M} = 3\pi\,\frac{2F_\infty}{(\frac{3}{2}\Omega)^2}\,\frac{H}{L} \tag{7}$$

Furthermore,

$$F_\infty f(z) = -\frac{16\sigma_{\mathrm{SB}} T^{3-b}}{3\kappa_0 \rho^{1+a}}\,\frac{\mathrm{d}T}{\mathrm{d}z} \tag{8}$$

[1] Next, replacing $\mathrm{d}T/\mathrm{d}z \to T/H$ and $T \to T_{\max}$ and using $(\gamma - 1)c_{\mathrm{p}}T_{\max} = c_{\mathrm{s}}^2 = \Omega^2 H^2$ we have

$$F_\infty f(z) \propto \frac{16\sigma_{\mathrm{SB}} H^{7-2b}}{3\kappa_0 \rho^{1+a}} \tag{10}$$

$$F_\infty f(z) \propto \frac{16\sigma_{\mathrm{SB}} H^{7-2b}}{3\kappa_0 (\rho H)^{1+a}} H^{1+a} \tag{11}$$

$$F_\infty f(z) \propto \frac{16\sigma_{\mathrm{SB}} H^{7-2b}}{3\kappa_0 \Sigma^{1+a}} H^{1+a} \tag{12}$$

$$F_\infty f(z) \propto \frac{16\sigma_{\mathrm{SB}} H^{8-2b+a}}{3\kappa_0 \Sigma^{1+a}} \tag{13}$$

$$\frac{4\sigma_{\mathrm{SB}} T^{4-b}}{3\kappa_0 \Sigma^{1+a}} H^a = \nu_{\mathrm{t}} \Sigma (\tfrac{3}{2}\Omega)^2 \tag{14}$$

2 Optically thin case

In the optically thin case, we have $T = \mathrm{const} \propto \dot{M}^{1/4}$. Furthermore, from $\dot{M} \propto \Sigma H^2$, and since $H^2 \propto T$ we have $\dot{M}^{3/4} \propto \Sigma$ or $\dot{M} \propto \Sigma^{4/3}$.

3 Disk parameters

factor 2?

$$\alpha_{\mathrm{SS}} = \dot{M}/3\pi\ \Sigma\Omega H^2 \tag{15}$$

$$\sigma_{\mathrm{SB}} T_{\mathrm{eff}}^4 = (\dot{M}/3\pi)\,(\tfrac{3}{2}\Omega)^2 \tag{16}$$

and

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[1] The following rewrite *may* be useful:

$$F_\infty f(z) = -\frac{16\sigma_{\mathrm{SB}}}{3\kappa_0}\left(\frac{T^n}{\rho}\right)^{1+a}\frac{\mathrm{d}T}{\mathrm{d}z} \tag{9}$$

where $n = (3-b)/(1+a)$ is the polytropic index.

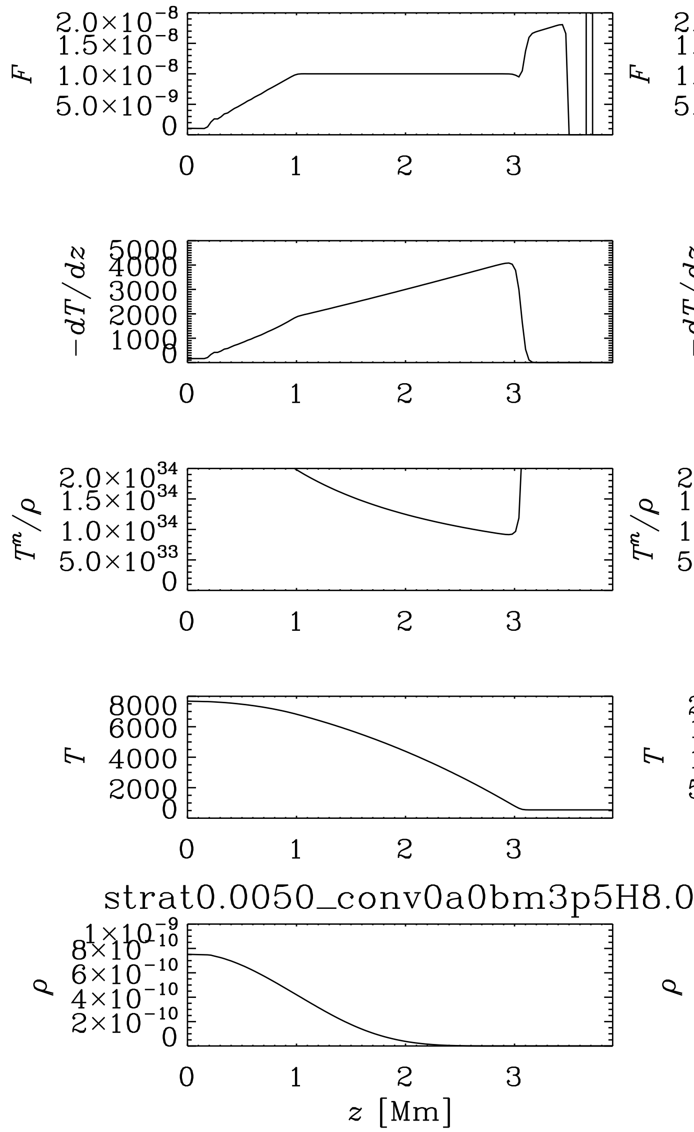

Figure 1: conv0a0bm3p5H8.0

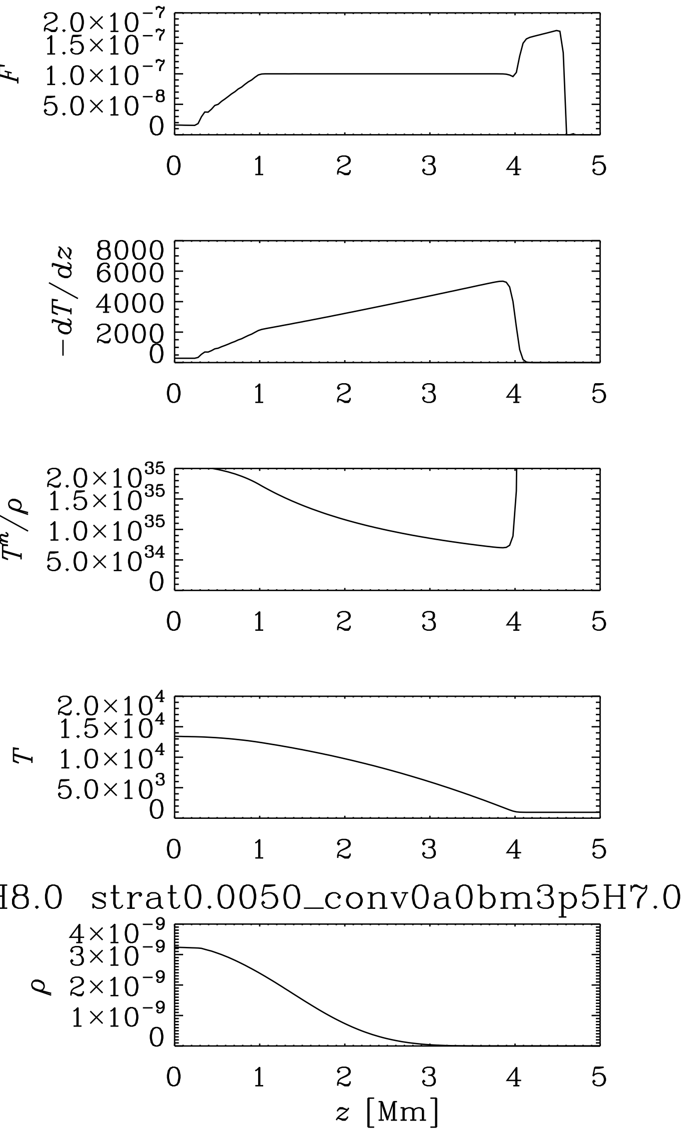

Figure 2: conv0a0bm3p5H7.0

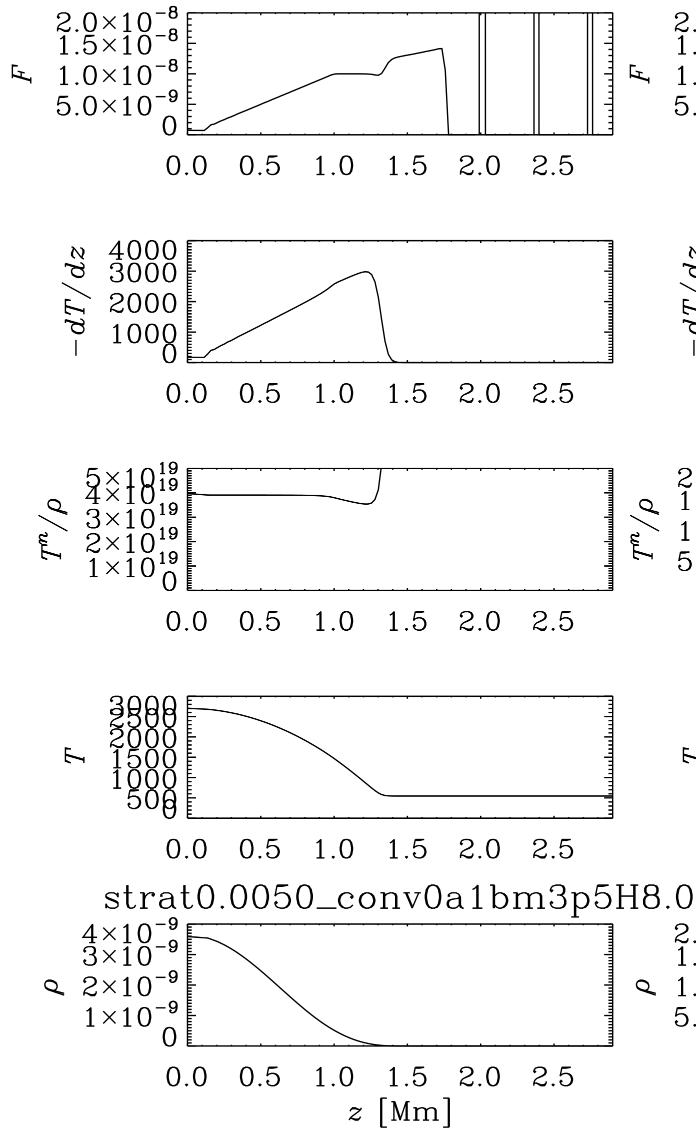

Figure 3: conv0a1bm3p5H8.0

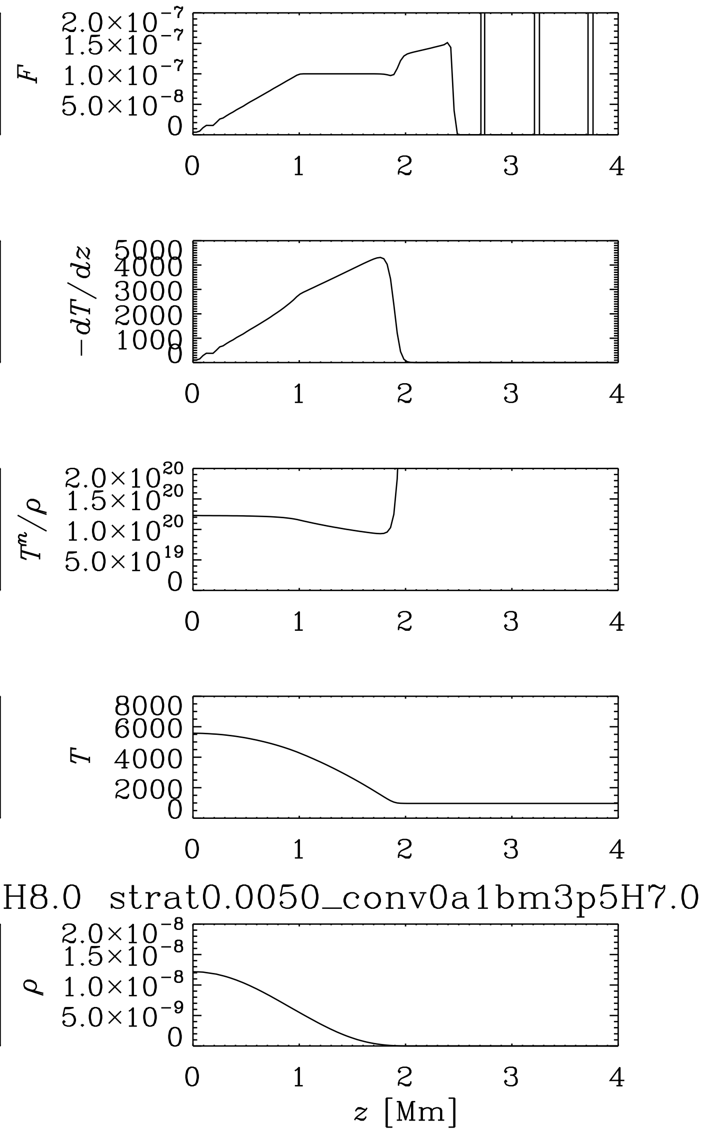

Figure 4: conv0a1bm3p5H7.0