Magnetic helicity flux in the Roberts flow

August 26, 2019, Revision: 1.9

1 Introduction

Magnetic helicity fluxes are believed to play a crucial role in astrophysical dynamos, when the magnetic Reynolds numbers are very large. Without magnetic helicity fluxes, the large-scale dynamo would evolve on a catastrophically slow, resistive timescale, as was demonstrated using simulations in periodic domains (Brandenburg, 2001). This behavior can be understood as a consequence of magnetic helicity (Field & Blackman, 2002), which can only change through microphysical resistivity. The hope was therefore that nonperiodic boundaries would suffice to allow for the dynamo to evolve on a faster timescale. This idea goes back to the work of Blackman & Field (2000), who showed that the α effect in mean-field electrodynamics (Krause & Rädler, 1980) can evade catastrophic quenching only when the dynamo is allowed to shed preferentially small-scale magnetic helicity. This was first formulated in the work of Kleeorin & Ruzmaikin (1982), using an earlier finding of Pouquet et al. (1976) that the total α effect is the sum of a contribution proportional to the kinetic helicity and another proportional to the current helicity. The latter quenches the former if the system attains a sufficient amount of small-scale magnetic helicity.

Subsequent simulations of dynamos with boundary conditions that permit a magnetic helicity flux have demonstrated that there is indeed a certain amount of magnetic helicity flux, but most of it is carried by the large-scale magnetic field and not, as was hoped, by the small-scale magnetic field. To understand the driving of magnetic helicity fluxes, we adopt a simple kinematic flow pattern and study the spreading of an initially localized magnetic field.

2 The model

We adopt a flow geometry where we can control separately the vertical and horizontal components. The initial magnetic field can be advected or even amplified by the flow. Having in mind the transport of magnetic helicity through some surface, we assume the magnetic field to be helical and confined to a layer of finite thickness. Specifically, we adopt an Arnold–Beltrami–Childress (ABC) field with an envelope in the z direction, i.e.,

$$\boldsymbol{B}(\boldsymbol{x},0)=f(z)\begin{pmatrix}A\sin kz+C\cos ky\\ B\sin kx+A\cos kz\\ C\sin ky+B\cos kx\end{pmatrix}, \qquad (1)$$

where $f(z)$ is a smoothed version of a tophat function with

$$f=\begin{cases}1 & \text{if } |kz|<\pi,\\ 0 & \text{otherwise.}\end{cases} \qquad (2)$$

The velocity is given by a modified Roberts flow of the form

$$\boldsymbol{U}=\frac{U_0}{2k_\mathrm{f}}\left(\cos\chi\boldsymbol{\nabla}\times\psi\hat{\boldsymbol{z}}+k_\mathrm{f}\sin\chi\tilde{\psi}\hat{\boldsymbol{z}}\right). \qquad (3)$$

where $k_\mathrm{f}=\sqrt{2}k$ is the effective wavenumber of the flow. We choose

$$\psi(x,y;\phi)=\cos(kx+\phi)\cos(ky+\phi), \qquad (4)$$

so the flow is two-dimensional and depends only on x and y. This flow consists of a component with a purely circular horizontal flow pattern proportional to $\boldsymbol{\nabla}\times\psi\hat{\boldsymbol{z}}$ and a perpendicular component along the z direction with the same horizontal pattern, which determines the locations of positive and negative values of u_z; see Figure 1 for a sketch.

The mixing angle χ controls the relative importance of the vertical and horizontal components. For $\chi=45^\circ$, we have equally strong horizontal and vertical flows. For $\chi=0^\circ$, the flow only has a horizontal circular component, while for $\chi=90^\circ$, the flow is purely vertical. The parameters of the model thus include A, B, C, χ, and ϕ.

In Fig. 1 we compare cases with $\chi=45^\circ$ and 90°. We see that the oppositely signed magnetic

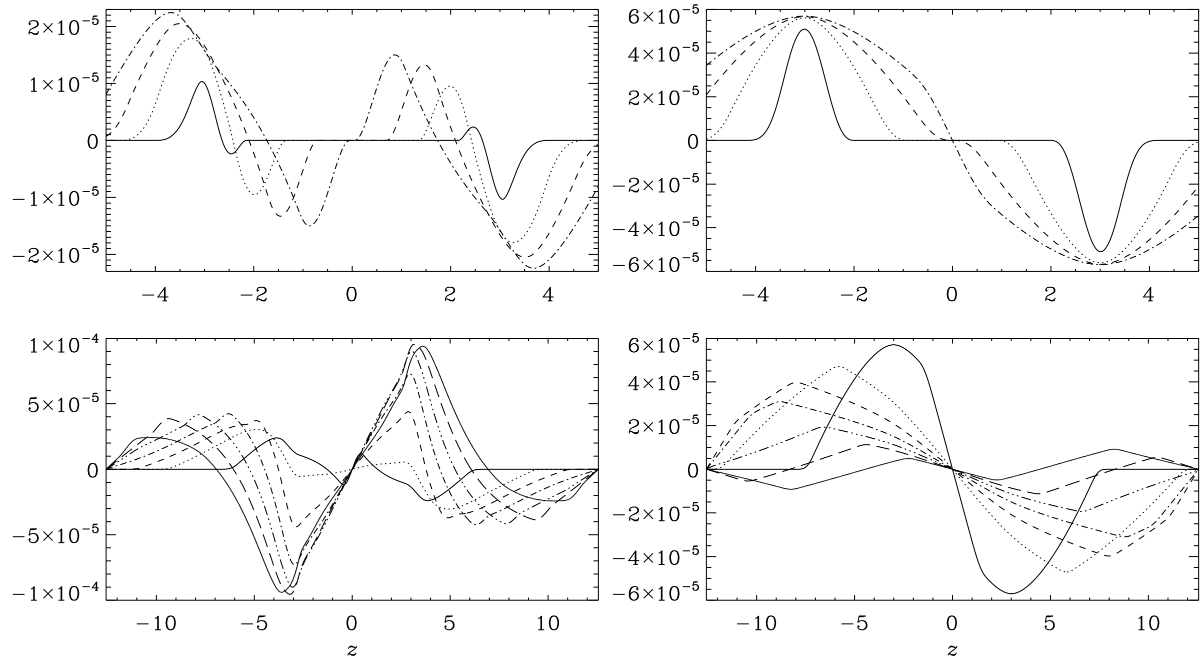

Figure 1: Left (`pexamz3b_256a`, $\chi = 45°$): Upper panel: $t = 0.3$, 0.9, 1.4, 1.8; lower panel: $t = 2.3$, 4.6, 5.7, 6.9, 8.0, 9.1, and 10.3. Right (`pexamz3b_256b`, $\chi = 90°$, i.e., advection only in the z direction): Upper panel: $t = 0.2$, 0.6, 0.9, 1.2; lower panel: $t = 1.6$, 3.1, 3.9, 4.7, 5.5, 6.2, and 7.0.

helicity flux in the inner parts are the result dynamo action by the helical Roberts flow. In the case with $\chi = 90°$, we see that the spreading of the magnetic helicity flux to the right (left) is symmetric at early times.

In Fig. 2 we see that the sign of χ, which determines the sign of the helicity of the flow, changes the overall sense of propagation of the helical field into the exterior.

In we compare cases with $\delta = 90°$, corresponding to Roberts flow-II with zero pointwise helicity, but opposite signs of the pumping direction. For positive (negative) values of χ, there is a positive (negative) magnetic helicity flux for $z > 0$. to the right

The induction equation for the magnetic field, $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$, is solved in terms of the magnetic vector potential $\boldsymbol{A}$ and obeys

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{U} \times \boldsymbol{B} + \eta \nabla^2 \boldsymbol{A} \qquad (5)$$

where η is the magnetic diffusivity. Its value is given in terms of the magnetic Reynolds number,

$$\mathrm{Re} = U_0 / k\eta. \qquad (6)$$

We choose the side length of the domain to be $L = 2\pi$, so that $k = 2\pi/L = 1$. We define mean fields through xy averaging, e.g.,

$$\overline{\boldsymbol{A}}(z,t) = \int \boldsymbol{A}(x,y,z,t)\,\mathrm{d}x\,\mathrm{d}y/L^2. \qquad (7)$$

We determine the evolution of the mean magnetic helicity density $h = h_\mathrm{m} + h_\mathrm{f}$, where $h_\mathrm{m} = \overline{\boldsymbol{A}} \cdot \overline{\boldsymbol{B}}$ and $h_\mathrm{f} = \overline{\boldsymbol{a} \cdot \boldsymbol{b}}$ are the contributions from the mean and fluctuating parts. Likewise, the magnetic helicity flux is

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bihelical fields, k=1,5.
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References

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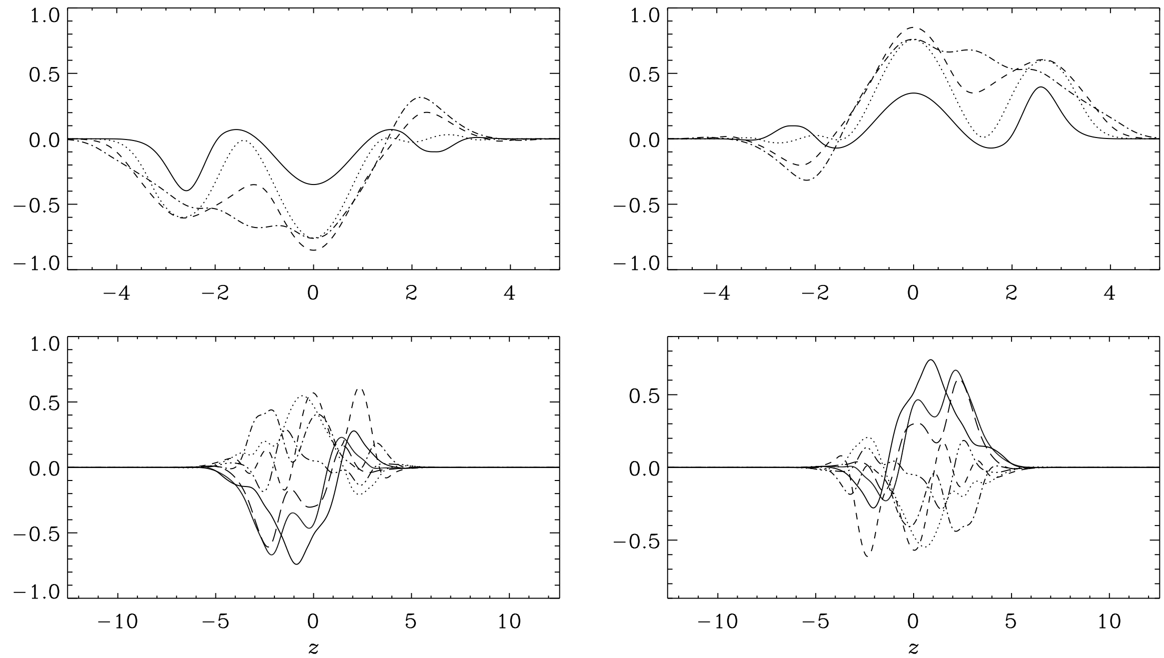

Figure 2: p/m-run with $\chi = \pm 30°$; upper panel: $t = 0.2$, 0.6, 0.9, 1.2; lower panel: $t = 1.6$, 3.1, 3.9, 4.7, 5.5, 6.2, and 7.0.

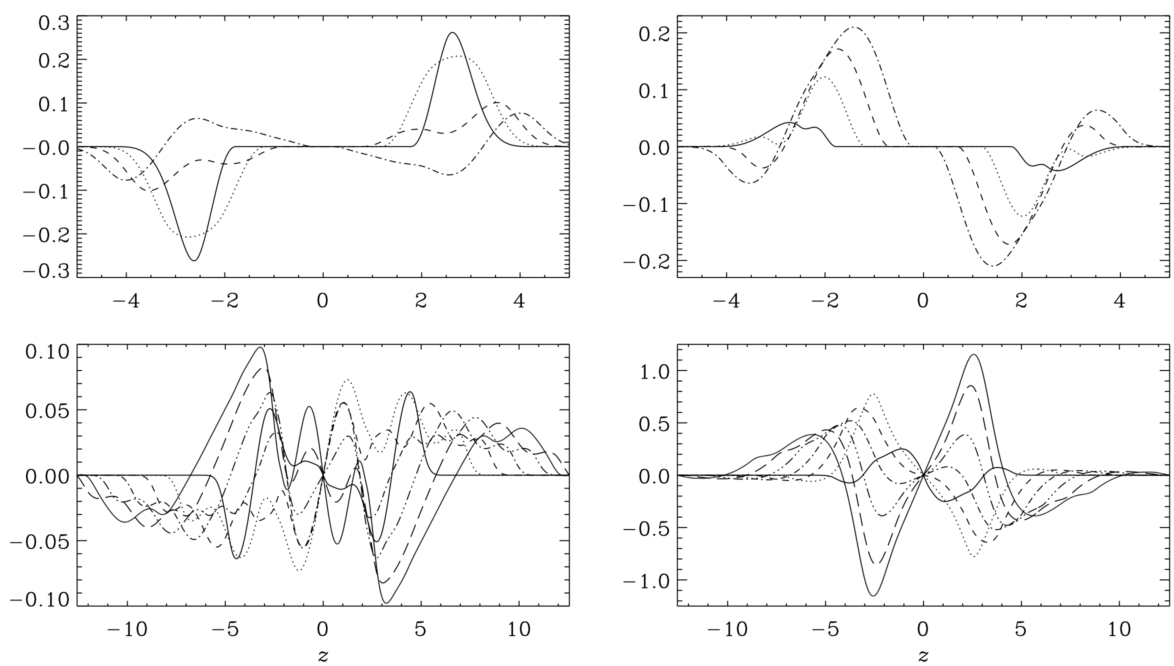

Figure 3: p/m-run with $\chi = \pm 30°$ and $\delta = 90°$; upper panel: $t = 0.2$, 0.6, 0.9, 1.2; lower panel: $t = 1.6$, 3.1, 3.9, 4.7, 5.5, 6.2, and 7.0.

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