1. **Blast wave in 2-D.** Use dimensional arguments to find the blast wave solution for the 2-dimensional case. Assume that the solution depends on the 2-dimensional surface (or column) density \( \Sigma = \int \rho \, dz \), instead of \( \rho \). Hint: What are the dimensions of \( \Sigma \)?

2. **Balbus-Hawley instability.** The Balbus-Hawley instability is governed by the equations

\[
-i \omega \hat{v}_x - 2 \Omega \hat{v}_y = \frac{B_0}{\rho_0 \mu_0} i k \hat{B}_x,
\]

\[
-i \omega \hat{v}_y + \frac{1}{2} \Omega \hat{v}_x = \frac{B_0}{\rho_0 \mu_0} i k \hat{B}_y,
\]

\[
-i \omega \hat{B}_x = B_0 i k \hat{v}_x,
\]

\[
-i \omega \hat{B}_y = B_0 i k \hat{v}_y - \frac{3}{2} \Omega \hat{B}_x.
\]

Show that the eigenfunction is given by

\[
\begin{pmatrix}
\hat{v}_x \\
\hat{v}_y \\
\hat{B}_x \\
\hat{B}_y
\end{pmatrix} = \begin{pmatrix}
\frac{\omega^2 - v_A^2 k^2}{2 \Omega} \\
\frac{-ik B_0}{\omega^2 - v_A^2 k^2 - \Omega^2} \\
\frac{i k B_0}{\omega^2 - v_A^2 k^2 - \Omega^2} \\
\frac{i k B_0}{\omega^2 - v_A^2 k^2 - \Omega^2}
\end{pmatrix}
\]

3. **Jeans instability with rotation.** In the presence of rotation one has to take the Coriolis force into account. The Euler equations takes then the following form

\[
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla p - 2 \Omega \times \mathbf{v} - \nabla \Phi,
\]

where \( \Omega = (0,0,\Omega) \) is the rotation vector and \( \Omega \) is the (constant) rotation rate. Find the dispersion relation \( \omega = \omega(k) \) assuming an isothermal equation of state, i.e. \( p/\rho = c_s^2 = \text{const.} \). Hint: write down the equations for all 3 components of \( \mathbf{v} \) and the other 2 variables in matrix form (5 x 5 matrix!) and find where the determinant is zero.

4. **Work against the Lorentz force.** Show that

\[
\mathbf{J} \cdot \mathbf{E} = \mathbf{v} \cdot (\mathbf{J} \times \mathbf{B}),
\]

where \( \mathbf{E} = -\mathbf{v} \times \mathbf{B} \).

5. **Hydromagnetic equations in conservative form.** Use the continuity, Euler, and energy equations together with the induction equation to derive the following equations

\[
\frac{\partial}{\partial t} (\rho v_i) = -\frac{\partial}{\partial x_j} \left[ \rho v_i v_j + \delta_{ij} p + \frac{1}{\mu_0} (B_i B_j - \frac{1}{2} \delta_{ij} B^2) \right]
\]

\[
\frac{\partial}{\partial t} \left( \frac{1}{2} \rho v^2 + \rho e + \frac{1}{2 \mu_0} B^2 \right) = -\nabla \cdot \left[ \mathbf{v} \left( \frac{1}{2} \rho v^2 + \rho e + p \right) + \mathbf{E} \times \mathbf{B} \right]
\]
6. **Alfvén speed.** Estimate the Alfvén speed \( v_A = B/\sqrt{\rho \mu_0} \) for the interstellar medium. Assume \( B = 3 \mu G \) (1 G = 10\(^{-4}\) Tesla) and \( \rho = 2 \times 10^{-24} \text{g/cm}^3 \). Carry out the calculation using both gaussian and SI units and make sure the result is the same!

7. **Advection test.**
   Use the **Pencil Code** to simulate the advection of a passive scalar obeying the equation
   \[
   \frac{dc}{dt} = -U \frac{dc}{dx} + \kappa \frac{d^2c}{dx^2}
   \]
   where \( U = \text{const} \) is a parameter. Use a smoothed hat function as initial condition. You may choose
   
   \begin{verbatim}
   &pscalar_init_pars
   initlncc='hatwave-x', ampllncc=1e-0, widthcc=.1
   /
   \end{verbatim}
   
   (a) Determine values of `pscalar_diff` for which the Gipps phenomena are kept at a minimum.
   (b) Study how this depends on the width parameter of the initial profile, `widthcc=.1`.
   (c) How does the run time affect the results?

8. **Effective wavenumbers.**
   (a) Show that \( \nabla \times A = kA \) for \( A = (0, \sin kx, \cos kx) \), and explain why this allows you to calculate an effective wavenumber.
   (b) Use the **Pencil Code** from [http://pencil-code.googlecode.com](http://pencil-code.googlecode.com) to calculate effective first and second wavenumbers. The trick is to use just the induction equation to produce an initial Beltrami field, i.e.
   \[
   \text{HYDRO} = \text{nohydro}
   \text{DENSITY} = \text{nodensity}
   \text{MAGNETIC} = \text{magnetic}
   \text{VISCOSITY} = \text{noviscosity}
   \text{EOS} = \text{noeos}
   \]
   So we don’t use any hydro or density, etc. The Beltrami field is then initialized in `start.in` using:
   
   \begin{verbatim}
   &magnetic_init_pars
   initaa='Beltrami-x', amplaa=1., kx_aa=16.
   /
   \end{verbatim}
   
   Finally, you need to output the relevant material in the ’print.in’ file, i.e.
   
   \begin{verbatim}
   abm(f20.14)
   ajm(f20.14)
   arms(f10.6)
   /
   \end{verbatim}
   Compile the code in one dimension and run the code for one timestep.
   (c) Determine \( \langle A \cdot B \rangle / \langle A^2 \rangle \) and \( \langle A \cdot J \rangle / \langle A^2 \rangle \) for different wavenumbers of the Beltrami field.