When do SNR overlap & collectively inflate a SB?

i) $t_{SF} \gg \Delta t_{SN}$, $R_{sn}(t_{SF}) > R_{cluster}$ — energy

ii) $t_{stall} > \Delta t_{SN}$, $R_{sn}(v_{sn} = \delta v) > R_{cluster}$ — momentum

iii) $t_{merge} > \Delta t_{SN}$, $R_{merge} > R_{cluster}$ — Cioffi & McKee 1988

i) When does cooling kick in & a shell form? $t_{SF}$

in Sedov-Taylor Phase: $E = \rho R_{ST}^3 (R_{ST}^2/t^2)$

$$R_{ST} \simeq (E t^2/\rho)^{1/5}$$

$$v_{ST} \simeq \frac{2}{5} R_{ST}/t = \frac{2}{5} (E/\rho t^3)^{1/5}$$

$$T_{ST} \simeq \frac{3}{16} \frac{\mu m_p v_{ST}^2}{k_B}$$

Cooling becomes important when $T \lesssim 10^6$ K ($5 \times 10^5$ K)

$$\Rightarrow t_{SF} \sim 4 \times 10^4 \text{ yr } E_{51}^{\frac{1}{5}} n_0^{-\frac{1}{2}} \quad \text{(Kim \& Ostriker 2015)}$$

$$R_{SF} \sim 25 \text{ pc } E_{51}^{1/3} n_0^{-2/5}$$

ii) when does an individual SNR stall (i.e. $v_{sn} = \delta v$)?

$$P_{SN} = \rho R^3 R/t$$

$$\Rightarrow R_{mom} = \left(\frac{P_{SN} t}{\rho}\right)^{1/4} \quad v_{mom} = \frac{1}{4}\left(\frac{P_{SN}}{\rho t^3}\right)^{1/4}$$

$t_{stall}$ when $v_{mom} = \delta v$

$$\Rightarrow t_{stall} = \left(\frac{P_{SN}}{\rho (4\delta v)^4}\right)^{1/3} \approx 2 \text{ Myr} \left(\frac{P_{SN}}{3\times10^5}\right)^{\frac{1}{3}} n_0^{-\frac{1}{3}} \delta v_{10}^{-\frac{4}{3}}$$

$$R_{stall} = \left(\frac{P_{SN}}{\rho}\right)^{\frac{1}{3}} \frac{1}{(4\delta v)^{1/3}} \approx 70 \text{ pc } P_{SN,5.5}^{\frac{1}{3}} n_0^{-\frac{1}{3}} \delta v_{10}^{-\frac{1}{3}}$$

iii) More careful version of (ii) $t_{merge} = 2 \text{ Myr } n_0^{-\frac{18}{49}} \delta v_{10}^{-10/7}$

$$R_{merge} = 70 \text{ pc } n_0^{-0.12} \delta v_{10}^{-10/7}$$

for overlap requirements

if (i) need $\Delta t_{SN} < t_{SF}$

$$\Delta t_{SN} = \frac{t_{SN}}{N_{SN}} = \frac{t_{SN}}{M_{cl}/m_*} \Rightarrow M_{cl} > m_* \frac{t_{SN}}{t_{SF}}$$

$$\Rightarrow M_{cl} > 7.5 \times 10^4\, M_\odot\, E_{51}^{-\frac{1}{5}}\, n_0^{\frac{1}{2}}$$

$$\Rightarrow \epsilon_* > 0.6\, E_{51}^{-1/5}\, h_{100}^{-5/2}\, \Sigma_{100}^{-1/2}$$

if (ii) need $\Delta t_{SN} < t_{stall}$

$$\Rightarrow M_{cl} > 1500\, M_\odot\, n_0^{\frac{1}{3}}\, \delta v_{10}^{\frac{4}{3}}\, P_{SN,5.5}^{-\frac{1}{3}}$$

$$\epsilon_* > 0.006\, h_{100}^{-7/2}\, \Sigma_{100}^{-\frac{2}{3}}\, \delta v_{10}^{4/3}\, P_{SN,5.5}^{-\frac{1}{3}}$$

easy!! $\Rightarrow$ SNR overlap!!

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All of this under the assumption that stars form in a cluster of mass $M_{cl}$

$$M_{cl} = \epsilon_* M_{GMC} = \epsilon_* \pi h^2 \Sigma_{gas}$$

1 SN per $m_*$ formed $\Rightarrow N_{SN} = \frac{M_{cl}}{m_*}$

SN go off uniformly spaced over $t_{SN} \sim 30$ Myr $\Rightarrow \Delta t_{SN} = \frac{t_{SN}}{N_{SN}}$

$$\dot{E}_{SN} = \frac{E_{SN}}{\Delta t_{SN}} = \frac{E_{SN}}{t_{SN}} \frac{m_*}{M_{cl}} = \frac{E_{SN}}{t_{SN}} \frac{m_*}{\epsilon_* \pi h^2 \Sigma_{gas}} \quad \& \quad \dot{P}_{SN} = \frac{P_{SN}}{t_{SN}} \frac{m_*}{\epsilon_* \pi h^2 \Sigma_{gas}}$$

in momentum driven SB evo: $\frac{d}{dt}\left(M \frac{dR}{dt}\right) = \dot{P}$

$$\Rightarrow R_{sh} \approx \left(\frac{\dot{P}_{SN}}{\rho}\right)^{\frac{1}{4}} t^{1/2} \quad \& \quad v_{sh} \approx \frac{1}{2} \frac{(\dot{P}_{SN}/\rho)^{\frac{1}{2}}}{R_{sh}}$$

$$\dot{P}_{SN} = \frac{P_{SN}}{\Delta t_{SN}}, \quad \Delta t_{SN} = \frac{N_{SN}}{t_{SN}}, \quad N_{SN} = \frac{M_{cluster}}{m_*} = \frac{\epsilon \pi h^2 \Sigma}{m_*}, \quad \rho \approx \frac{\Sigma}{2h}$$

① does bubble reach $h$ before SNe run out $\Rightarrow R_{sh} = h$ @ $t_{BO} < t_{SN}$

$$h = \left(\frac{\dot{P}_{SN}}{\rho}\right)^{\frac{1}{4}} t_{BO}^{1/2} \qquad \frac{\dot{P}_{SN}}{\rho} = \frac{\epsilon h^2 \Sigma}{m_*} \frac{P_{SN}}{t_{SN}} \Big/ (\Sigma/h) = \frac{\epsilon_* P_{SN}}{m_*} \frac{h^3}{t_{SN}}$$

$$\Rightarrow \frac{t_{BO}}{t_{SN}} = \left(\frac{m_*}{\epsilon P_{SN}} \frac{h}{t_{SN}}\right)^{\frac{1}{2}} f_v^{\frac{1}{2}}, \quad f_v = \frac{n_{median}}{\langle n \rangle}$$

② has bubble stalled prior to reaching $h$? $v_{sh}(R_{sh} = h) > \delta v$

$$v_{sh}(h) = \frac{(\dot{P}_{SN}/\rho)^{\frac{1}{2}}}{h} = \frac{\left(\frac{\epsilon_* P_{SN}}{m_*} \frac{h^3}{t_{SN}}\right)^{\frac{1}{2}}}{h} = \left(\frac{\epsilon_* P_{SN}}{m_*} \frac{h}{t_{SN}}\right)^{\frac{1}{2}} f_v^{-\frac{1}{2}}$$

$$\frac{h}{t_{SN}} = 3\ \mathrm{km/s} \left(\frac{h}{100\ \mathrm{pc}}\right)\left(\frac{t_{SN}}{30\ \mathrm{Myr}}\right)^{-1} \quad \& \quad \frac{\epsilon_* P_{SN}}{m_*} = 30\ \mathrm{km/s} \left(\frac{\epsilon_*}{0.01}\right)\left(\frac{P_{SN}/m_*}{3 \times 10^3\ \mathrm{km}}\right.$$

$$\Rightarrow \frac{t_{BO}}{t_{SN}} \approx 0.3 \quad \& \quad v_{sh}(h) \approx 10\ \mathrm{km/s}\ h_{100}^{\frac{1}{2}}\ \epsilon_{0.01}^{\frac{1}{2}}$$

$\epsilon_{crit}$ s.t. $v_{sh}(h) = \delta v$

$$\epsilon_{crit} = \frac{\delta v^2 f_v}{\left(\frac{P_{SN}}{m_*}\right)\left(\frac{h}{t_{SN}}\right)} \sim 1\% \quad (\text{with constants} \sim 1.5 - 2\%)$$

Implications of Breakout (or not).

1) if bigger GMCs are harder to disrupt then $\epsilon_* \uparrow$ as $\Sigma_g \uparrow$ since $M_{GMC} \sim h^2 \Sigma_g$

$$\epsilon_*(\Sigma_g) = 100\% \ (\Sigma_g / 3000\ M_\odot/pc^2)$$

$$\Rightarrow \frac{\Sigma_{crit}}{3000\ \frac{M_\odot}{pc^2}} = \epsilon_{crit} = \underbrace{0.01\ \delta v_{10}^2\ h_{100}^{-1}}_{\text{breakout — overlap is 2x easier}}$$

$$\text{HSE} \Rightarrow \frac{dP}{dz} = \rho g \Rightarrow \frac{\rho \delta v^2}{h} = \rho \frac{\partial \phi}{\partial z} = \rho \frac{\partial \phi}{\partial r}\frac{\partial r}{\partial z} = \rho \frac{V_c^2}{r}\frac{h}{r}$$

$$\Rightarrow h = \delta v \frac{r}{V_c} = \delta v\, t_{dyn}$$

$$\Rightarrow \Sigma_{crit} = 30\ \frac{M_\odot}{pc^2}\ \delta v_{10} \left(\frac{t_{dyn}}{10\ Myr}\right)^{-1} \quad \text{where } \frac{2\ kpc}{200\ km/s} = 10.$$

Breakout if $\Sigma_{gas} \gg \Sigma_{crit}$

using KS relation we can convert to $\dot{\Sigma}_*$ criterion

$$\dot{\Sigma}_* \gg 0.03\ \frac{M_\odot}{yr\ kpc^2}$$

Note Heckman (2002) finds threshold for winds at $\dot{\Sigma}_* = 0.1\ M_\odot/yr/kpc^2 \sim 3\ \Sigma_{SFR,crit}$

2) Modify KS — $\Sigma_{SFR} \langle \frac{P_{SN}}{m_*} \rangle = G\, \Sigma_{disk} \Sigma_{gas}$

$$\langle \frac{P}{m_*} \rangle = \frac{1}{M_{cl}} \int_0^{t_{bo}} \dot{P}\, dt = \frac{1}{M_{cl}} \left( P_{SN} \dot{N}_{SN} t_{BO} \right) = \frac{1}{M_{cl}} \frac{P_{SN} M_{cl}}{t_{SN} M_*} t_{BO} = \frac{P_{SN}}{M_*} \frac{t_{BO}}{t_{SN}}$$

if $t_{bo} > t_{SN}$ then $\langle \frac{P}{m_*} \rangle = \frac{P_{SN}}{m_*}$ else $\langle \frac{P}{m_*} \rangle < \frac{P_{SN}}{m_*}$

Energy conservation: $\frac{\Sigma_{SFR}}{h}\left\langle\frac{P_{SN}}{m_*}\right\rangle \delta v = \rho\frac{\delta v^3}{h} = \frac{\Sigma_{gas}\,\delta v^3}{h^2}$

Vertical HSE : $\frac{\partial P}{\partial z} \sim \rho\frac{\delta v^2}{h} = \rho g = \rho\frac{GM}{r^2} = \rho G\Sigma_{disk}$

$\Rightarrow \delta v^2 = G\Sigma_{disk}\, h$ $\quad f_{gas} = \frac{\Sigma_{gas}}{\Sigma_{disk}} \Rightarrow f_{gas}\,\delta v^2 = G\Sigma_{gas}\, h$

$\Rightarrow \Sigma_{SFR}\left\langle\frac{P_{SN}}{m_*}\right\rangle = \frac{f_{gas}\,\delta v^4}{G h^2} = \Sigma_{SFR}\left(\frac{1}{\epsilon_*}\frac{P_{SN}}{m_*}\frac{h}{t_{SN}}\right)^{\frac{1}{2}}$

$\Rightarrow \frac{f_{gas}^2\,\delta v^8}{G^2 h^4} = \Sigma_{SFR}^2\,\frac{P_{SN}}{\epsilon_* m_*}\frac{h}{t_{SN}} \Rightarrow h = \left(\frac{f_{gas}^2\,\delta v^8\,\epsilon_* m_* t_{SN}}{G\,\Sigma_{SFR}^2\,P_{SN}}\right)^{\frac{1}{5}}$

or $\Sigma_{SFR}\left\langle\frac{P_{SN}}{m_*}\right\rangle = G\Sigma_{gas}^2/f_{gas}$

if $t_{BO} < t_{SN}$ $\quad \Sigma_{SFR}\left(\frac{1}{\epsilon_*}\frac{P_{SN}}{m_*}\frac{h}{t_{SN}}\right)^{\frac{1}{2}} = G\Sigma_{gas}^2/f_{gas} \Rightarrow \frac{1}{\epsilon}\frac{P_{SN}}{m_*}\frac{h}{t_{SN}} = \frac{G^2\Sigma_g^4}{\Sigma_{SFR}^2 f_{gas}^2}$

$\Rightarrow h = \frac{G^2\Sigma_g^4}{\Sigma_{SFR}^2 f_{gas}^2}\,\frac{\epsilon_* m_* t_{SN}}{P_{SN}}$

let $\Sigma_{SFR} =$ (observationally motivated) $\quad \left(\frac{\Sigma_g}{\Sigma_0}\right)^{\alpha}\frac{\Sigma_0}{t_{dep0}} = \Sigma_{SFR}$

& $\epsilon_* = \epsilon_{*,0}\left(\Sigma_{gas}/\Sigma_0\right)^{\beta}$ (theoretically motivated)

then $h = \frac{G^2 m_* t_{SN}\,\Sigma_0^4\,\epsilon_{*0}}{f_{gas}^2\,P_{SN}/t_{dep0}^2}\left(\frac{\Sigma_{gas}}{\Sigma_0}\right)^{4-2\alpha+\beta}$

$h = 200\,pc\;\Sigma_{d,100}^2\left(\frac{\epsilon}{0.1}\right)\left(\frac{t_{dep}}{Gyr}\right)^2$

if $\alpha = 1$ & $\beta = 0$ then $h = G^2\Sigma_g^2\,\epsilon_0\frac{m_*}{P_{SN}}\frac{t_{SN}\,t_{dep0}^2}{f_{gas}^2} = (G\Sigma_d)^2\,\epsilon_0\frac{m_*}{P_{SN}}\,t_{SN}\,t_{dep0}^2$