KS relation: \[ \Sigma_{\text{eff}} \]

Equilibrium model:

\[
\begin{align*}
W & \propto G \Sigma_{\text{eff}} \Sigma_{\text{disc}} \\
\frac{P_{\text{turb}}}{h} & \propto \frac{\dot{E}_{\text{in}}}{C_T} \\
\end{align*}
\]

\[
\text{SF-driven turb:}
\]

\[
\dot{E}_{\text{in}} \propto \frac{\Sigma_{\text{SFR}} (P_\star/\text{mp})}{C_T} \\
\dot{E}_{\text{disc}} \propto \frac{P C_T}{h/C_T}
\]

\[ \Rightarrow P_{\text{turb}} \propto \Sigma_{\text{SFR}} (P_\star/\text{mp}) \]

\[ \Sigma_{\text{SFR}} \propto \frac{G}{P_\star/\text{mp}} \Sigma_{\text{disc}} \]

\[ \Sigma_{\text{eff}} \sim 1\% \]

\[ t_{\text{dpl}} = \frac{\Sigma_{\text{eff}}}{\Sigma_{\text{SFR}}} \sim 1 \text{ Gyr} \]
Accretion-driven turb (radial transport):
\[ \dot{e}_{\text{in}} \approx \frac{\dot{M}_{\text{acc}} V_c}{R^2 h} \]
\[ \approx \left(1 + n\right) \frac{\sum_{\text{SFR}} V_c}{h} \]
\[ \dot{M}_\ast \propto \dot{M}_{\text{acc}}/(1 + n) \]
\[ \Gamma_{\text{disc}}, n \text{ are the same as in SF-driven case, so obtain } K_S \text{ w/ } \left(P_\ast/M_\ast\right)_C \Rightarrow (1 + n), \]
\[ \left(P_\ast/M_\ast\right) \Rightarrow (1 + n)! \]
\[ \Rightarrow \sum_{\text{SFR}} \approx \frac{G \cdot C_T}{(1 + n) V_c^2} \sum_{z} \sum_{\text{disc}} \]
\[ Q \approx 1 \Rightarrow \Phi_g \approx C_T V_c \]
\[ \Rightarrow \sum_{\text{SFR}} \approx \frac{G \cdot f_g}{(1 + n) V_c} \sum_{z} \sum_{\text{disc}} \]

SF efficiencies (disc scalp):
\[ \varepsilon_{\text{eff}}^\text{disc} = \frac{\sum_{\text{SFR}}}{\sum_{\text{S}} t^\text{eff} = G \sum_{\text{disc}} t^\text{eff}} \]
\[ \propto \frac{C_T}{P_\ast/M_\ast} \]
SF case using: \( t^\text{eff} = 1/\sqrt{P_\ast} \), \( \bar{\rho} \propto \sum_{\text{disc}} / h \)
\( Q \approx V_c C_T \)
\( G \sum_{\text{disc}} R \)
\( \sim 1 \)

\[ J_{\text{Ne}}: P_\ast/M_\ast \sim 1,000 - 3,000 \text{ Km/s}. \]
\[ \Rightarrow \varepsilon_{\text{eff}}^\text{disc} \sim 1\% \text{ For } C_T \sim 10 - 30 \text{ Km/s} \]

Accr. case
\[ P_\ast/M_\ast \rightarrow (1 + n) V_c^2 \]
\[ \Rightarrow \varepsilon_{\text{eff}}^\text{disc} \propto \frac{1}{1 + n} \left(\frac{C_T}{V_c}\right)^2 \sim 1\% \]
\[ \sim 1 \text{ or } 0.1\% \]
randomly driven SNe

Assuming: SNe drive turb. on scale $R_{SN} = \text{radius when remnant vel.} = C_T$

Martizzi+15,16 showed: $C_T \approx 10 \text{Km/s} \left( \frac{f_g}{0.1} \right)^{-\frac{1}{15}} \left( \frac{\Sigma}{100 M_\odot/pc^2} \right)^{\frac{1}{15}}$, almost const.

If applies to real galaxies, accretion limit may be realized when $C_T \gtrsim 10 \text{Km/s}$

Disc wants to maintain $Q \sim 1$, $\Rightarrow C_T \approx f_g V_c$

$f_g \Rightarrow \text{grav. inst. drives turb.}$

Implications: $E_{\text{eff}}^\text{disc}$ largely decoupled from cloud-scale $E_{\text{eff}}^\text{cloud}$

"not universal (depends on galaxy properties)"

Questions: Can we unambiguously (in sims or obs.) determine SF vs. accretion contributions?

In SF case, is turb. excited by SNe in vol.-filling medium or GMC disruption?