High Energy Processes in Accreting Black Holes
ASP Conference Series, Vol. 161, 1999
Juri Poutanen and Roland Svensson, eds.

The Dynamics of Turbulent Viscosity

Ulf Torkelsson

Chalmers University of Technology/Göteborg University, Department of Theoretical Physics, Astrophysics Group, S-412 96 Gothenburg, Sweden

Gordon I. Ogilvie and James E. Pringle

Institute of Astronomy, Madingley Road, Cambridge CB3 0HA, United Kingdom

Axel Brandenburg

Department of Mathematics, University of Newcastle upon Tyne, NE1 7RU, United Kingdom

Åke Nordlund

Theoretical Astrophysics Center, Juliane Maries vej 30, DK-2100 Copenhagen Ø, Denmark

Robert F. Stein

Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, USA

Abstract. The first generation of numerical simulations of magnetohydrodynamic turbulence in accretion disks concentrated on determining the component of the stress tensor that is responsible for driving the accretion flow. However other components of the stress tensor are important, for instance for determining the response of the disk to a warping motion. We show how these effects can be studied within the shearing box approximation.

1. Introduction

It is now commonly believed that the turbulent viscosity in accretion disks is caused by a magnetohydrodynamical instability (e.g. Balbus & Hawley 1991). The properties of the turbulence have been explored in numerical simulations approximating a small part of the disk with a Cartesian shearing box (e.g. Hawley, Gammie & Balbus 1995, Matsumoto & Tajima 1995, Brandenburg et al. 1995, Stone et al. 1996). The turbulent viscosity is customarily described in terms of the parameter α_{SS} (Shakura & Sunyaev 1973), which parameterizes the viscous stress in terms of the pressure in the accretion disk. This prescription was intended to describe the radial transport of angular momentum through the disk, and it is not clear that it is relevant for the response of the disk to kinematical perturbations, such as warps. In this paper we present a preliminary investigation of that problem.

Pringle (1996) described how an accretion disk may be unstable to a warping motion through the irradiation of the outer parts of the disk by a source at its centre. Locally, one of the effects of the warp is to induce an epicyclic motion whose amplitude varies linearly with the distance from the midplane of the accretion disk. The amplitude and phase of the epicyclic response determine the evolution of the warp (Papaloizou & Pringle 1983, Papaloizou & Lin 1995).

While it is not possible to investigate the global warps in the shearing box approximation that has so far been used in the numerical simulations of turbulence in accretion disks, the interaction between the turbulence and the induced epicyclic motion can be studied. The epicyclic motion is itself a solution of the hydrodynamic equations in a laminar accretion disk.

We summarize the results of the numerical studies of the turbulence in an accretion disk in Sect. 2. Sect. 3 provides a short analytical discussion of the warps and the epicyclic motion in an accretion disk, as an introduction to the numerical simulations that we present in Sect. 4 (a more detailed report on the numerical simulations will appear elsewhere, Torkelsson et al. 1998, in preparation). Finally a brief discussion of the results are given in Sect. 5.

2. The Current Status of Magnetohydrodynamic Turbulence Simulations

The cause of the turbulent viscosity in accretion disks has been the major problem in the theoretical understanding of accretion disks for a very long time. It was commonly thought that the turbulence was induced by the differential rotation in the disk, although a Keplerian rotation profile is hydrodynamically stable according to Rayleigh's criterion. A major breakthrough was the realization that the corresponding magnetohydrodynamical flow is linearly unstable (Balbus & Hawley 1991) to an instability that had previously been described by Velikhov (1959) and Chandrasekhar (1960). Numerical simulations have since then shown that the instability produces turbulence in an accretion disk, and that the turbulent stresses transport the angular momentum outwards (e.g. Hawley et al. 1995, Matsumoto & Tajima 1995, Brandenburg et al. 1995, Stone et al. 1996).

The turbulence is dominated by the magnetic field in the sense that the energy density of the magnetic field exceeds that of the turbulent kinetic energy. However the velocity field shows more variations on small scales, and the forces are roughly comparable. The turbulent stresses do not vary much with the distance from the midplane of the accretion disk (Brandenburg et al. 1996), which shows that the viscosity prescription of Shakura & Sunyaev (1973) cannot be relied on for calculating the vertical structure of the accretion disk. The reason is that the Shakura & Sunyaev prescription makes the stress proportional to the pressure, which implies that the stresses should fall off with the pressure away from the midplane. A consequence of the strong stress in the surface layers is that those heat up the most and become hotter than the interior of the disk.

3. Epicyclic Motion

An epicyclic motion of the form

$$u_x = u_0\,(z)\cos\Omega_0 t, \tag{1}$$

$$u_y = -\frac{1}{2}u_0\,(z)\sin\Omega_0 t, \tag{2}$$

is a solution to the ideal hydrodynamic equations in a shearing box with the Keplerian angular velocity Ω_0. The amplitude of the motion, $u_0(z)$, is an arbitrary function of z. The viscous damping of the motion is described by

$$\frac{\partial u_x}{\partial t} = 2\Omega_0 u_y + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\nu\frac{\partial u_x}{\partial z}\right), \tag{3}$$

$$\frac{\partial u_y}{\partial t} = -\frac{1}{2}\Omega_0 u_x + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\nu\frac{\partial u_y}{\partial z}\right). \tag{4}$$

If we assume that the density, ρ, and the viscosity, ν, are constant, and that the amplitude of the velocity varies as $\sin kz$, the velocity will be damped as $e^{-\nu k^2 t}$. However, if we interpret the turbulent stresses described in Sect. 2 as a viscosity, then $\rho\nu$ will be independent of z, so that the damping will be the fastest at large $|z|$.

4. Simulations of a Decaying Epicyclic Motion

For our numerical simulations we started from a snapshot of a previous turbulence simulation. Our shearing box is located at a distance $R_0 = 10$ from a gravitating object with a mass such that $GM = 1$. The scale height of the medium in the box is 1, which yields the mean internal energy $7.4 \cdot 10^{-4}$. The size of the box is $L_x : L_y : L_z = 1 : 2\pi : 4$, where x and z vary between $\pm\frac{1}{2}L_x$ and $\pm\frac{1}{2}L_z$, respectively, and y goes from 0 to L_y. The box uses $31 \times 63 \times 63$ grid points. We employ a cooling function with a relaxation time scale of 1.5 orbital periods to prevent the box from heating up. The horizontal boundary conditions are (sliding)-periodic like in previous publications (Brandenburg et al. 1995). The vertical boundaries work as perfect conductors with respect to the magnetic field, and the velocity is stress-free on the same boundaries with no flow through them. We add a radial velocity with an amplitude that is proportional to $\sin(\frac{\pi}{2}\frac{z}{L_z})$ to the snapshot. This perturbation approximates the linear dependence on z of the epicyclic motion induced by a warp for small z and fulfills the stress-free boundary conditions at the surface.

When the amplitude of the velocity perturbation is small, corresponding to a Mach number of 0.38 based on the adiabatic sound speed, the epicyclic motion decays the fastest in the surface layers as expected if $\rho\nu$ is constant with height. In Fig. 1 we plot $u_0 = \sqrt{\langle u_x\rangle^2 + 4\langle u_y\rangle^2}$ as a function of z at different times. Here $\langle\rangle$ denotes horizontal averages. The typical time scale for the decay of the epicyclic motion is 18 orbital periods. Interpreted in terms of the Shakura & Sunyaev prescription $\nu = \alpha_{\rm SS} c_{\rm s} H$, where $c_{\rm s}$ is the isothermal sound speed, and

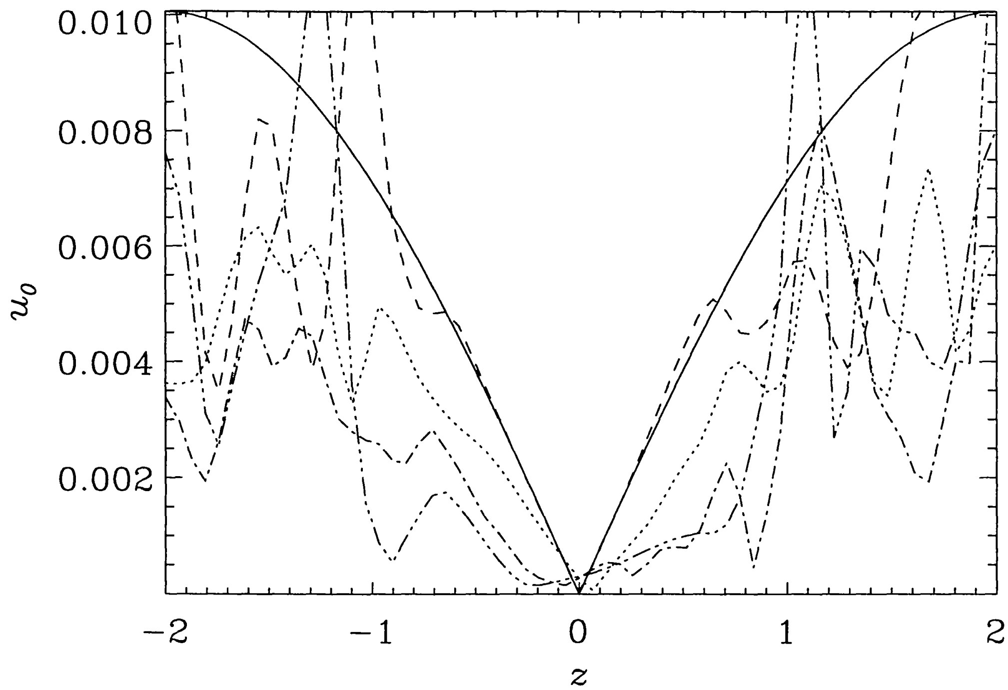

Figure 1. The time development of the amplitude of the epicyclic motion, u_0, as a function of z. The times are at the beginning of the simulation (solid line), after 5 orbital periods (dashed line), 15 orbital periods (dotted line), 25 orbital periods (dashed-dotted line), and 35 orbital periods (dashed- triple dotted line)

H is the Gaussian scale height, we get $\alpha_{SS} = 0.02$, which is a bit larger than the values we have previously obtained based upon the stresses in the simulations.

The picture changes significantly when the initial velocity perturbation is increased to a Mach number of 3.3. The damping time increases to some 25 orbital periods in the surface layers, and decreases to 14 orbital periods close to the midplane. The change in the damping time can be explained by the fact that the net magnetic stress is weakened in this simulation by the strong imposed motion. The weak stress can also account for the fact that the heating of the surface layers is sufficiently weak that they can relax towards the thermal state of the initial model. A weak magnetic stress does not necessarily imply a weak magnetic field, and the magnetic energy density shows a more complicated behavior. The magnetic field is getting weaker during the first 10 orbital periods of the simulation, but it is recovering during the last 20 orbital periods of the simulation, albeit without a corresponding recovery of the net magnetic stress. This variation can be attributed almost entirely to the variation of the azimuthal magnetic field, while the radial and vertical magnetic field components are decaying throughout the simulation.

5. Discussion

When interpreting the results in this paper it is important to keep in mind that the damping of the epicyclic motion is due to the rz- and ϕz-components of the turbulent stress, while the accretion is driven by the $r\phi$-component. Thus these simulations explore aspects of the turbulence that have not been studied hitherto. The value that we calculate for $\alpha_{\rm SS}$ from the decay of the epicyclic motion is larger than that we have previously calculated from the $r\phi$-stress, which suggests that the turbulent stress is anisotropic, but a more careful analysis is necessary to verify this.

For a Keplerian disk, as shown by Papaloizou & Pringle (1983), the amplitude of the epicyclic motion is controlled by the viscous response that we have measured in this paper. If $H/R < \alpha_{\rm SS} < 1$ the Mach number of the epicyclic motion will be

$$\mathrm{Ma} \sim \frac{A}{\alpha_{\rm SS}}, \tag{5}$$

and for $\alpha_{\rm SS} < H/R$

$$\mathrm{Ma} \sim \frac{A}{H/R}, \tag{6}$$

where $A = |\partial\beta/\partial \ln r|$ is the amplitude of the warp, as measured by the tilt angle β, and H/R is the relative thickness of the disk. This shows that even a weak warp may generate a strong epicyclic motion in a thin accretion disk. The supersonic régime of the epicyclic motion may be poorly described by existing linear theory, which makes the results of our simulations valuable.

The results for the subsonic epicyclic motion confirm the expectations that we have based upon previous simulations of turbulence in accretion disks. The supersonic epicyclic motion on the other hand generates some new features that we did not expect, but it is not clear to which extent these results apply on a warped accretion disk, as most of the new effects appear in layers in which the amplitude of the epicyclic motion depend only weakly on $|z|$. The magnetic stress is significantly smaller, and the disk is consequently heated less efficiently with a supersonic epicyclic motion. Further analysis is necessary to pinpoint the cause of this behavior. Some of the most obvious explanations are that either the strong epicyclic motion inhibits the dynamo effect that would otherwise generate the magnetic stress, or that the vertical shear associated with the epicyclic motion breaks up the turbulent eddies.

Acknowledgments. UT is supported by the Natural Sciences Research Council (NFR). Computer resources from the National Supercomputer Centre at Linköping University are gratefully acknowledged. This work was supported in part by the Danish National Research Foundation through its establishment of the Theoretical Astrophysics Center (ÅN). RFS is supported by NASA grant NAG5-4031.

References

Balbus, S. A., Hawley, J. F. 1991, ApJ, 376, 214

Brandenburg, A., Nordlund, Å., Stein, R. F., Torkelsson, U. 1995, ApJ, 446, 741

Brandenburg, A., Nordlund, Å., Stein, R. F., Torkelsson, U. 1996, in Basic physics of accretion disks, S. Kato et al. , Gordon & Breach, 285

Chandrasekhar, S. 1960, Proc. Nat. Acad. Sci., 46, 253

Hawley, J. F., Gammie, C. F., Balbus, S. A. 1995, ApJ, 440, 742

Matsumoto, R., Tajima, T. 1995, ApJ, 445, 767

Papaloizou, J. C. B., Lin, D. N. C. 1995, ApJ, 438, 841

Papaloizou, J. C. B., Pringle, J. E. 1983, MNRAS, 202, 1181

Pringle, J. E. 1996, MNRAS, 281, 357

Shakura, N. I., Sunyaev, R. A. 1973, A&A, 24, 337

Stone, J. M., Hawley, J. H., Gammie, C. F., Balbus, S. A. 1996, ApJ, 463, 656

Velikhov, E. P. 1959, Sov. Phys. JETP, 36, 1398