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LETTER

Letter to the Editor

Magnetic fields in young galaxies due to the cross-helicity effect

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Abstract. It is shown that the cross-helicity effect facilitates rapid growth of the large scale magnetic field in young galaxies. This field then acts as a seed for the standard $\alpha\Omega$-type dynamo at later stages. This mechanism may be responsible for the relatively strong magnetic fields observed in young high redshift galaxies.

Key words: magnetohydrodynamics – interstellar medium: magnetic fields – galaxies: magnetic fields – galaxies: evolution

1. Introduction

Observations of polarised synchrotron emission of high redshift galaxies have revealed the presence of microgauss magnetic fields (Kronberg et al. 1992). Conventional $\alpha\Omega$ dynamo theory may not be able to explain the amplification of weak seed magnetic fields $\sim 10^{-18}$ G to microgauss strengths after a few 10^9 years. The basic difficulty is that the growth rate of a large scale dynamo is typically some fraction ξ of the angular velocity Ω of the galaxy. Typical numbers are $\Omega = 30$ Gyr^{-1} and $\xi \approx 0.1 - 0.5$. Even in the most optimistic case, the amplification factor after $t = 1$ Gyr is just $\exp(0.5\Omega t) \approx 10^{6.5}$.

Several alternatives have been offered. Chiba & Lesch (1994) argue that both shear and radial compression during the early evolution of the galaxy could significantly amplify the magnetic field. However, doubt has been expressed as to whether the effect is strong enough and whether the neglect of turbulent magnetic diffusion is permissible (Beck et al. 1996).

Another more likely possibility is that a small scale dynamo could amplify the magnetic field on a short time scale (a few turnover times) and would then provide a strong initial magnetic field for the large scale (or mean-field) dynamo (Poezd et al. 1993, Beck et al. 1995, 1996). This initial small scale magnetic field has a typical scale of $l = 300$ pc and a strength of probably a few microgauss. Averaging in the toroidal direction at radius R over $N = 2\pi R/l \approx 100 - 1000$ cells, this initial field is weakened by a factor $1/\sqrt{N} = (3 - 10) \times 10^{-3}$. However, a large scale dynamo may then well be able to provide amplification of a few hundreds after about one Gyr (10^9 yr).

In recent years a new mechanism has been explored by Yoshizawa 1990, Yoshizawa & Yokoi 1993, and Yokoi 1996 who considered the transport properties of inhomogeneous turbulence by making use a two-scale direct-interaction approximation. They argued that the induction equation for the mean magnetic field $\boldsymbol{B}$ is supplemented by an inhomogeneous term proportional to the product of cross-helicity $\langle \boldsymbol{u}' \cdot \boldsymbol{b}' \rangle$ and mean vorticity. Here, $\boldsymbol{u}'$ and $\boldsymbol{b}'$ are the fluctuating components of velocity and magnetic field, respectively. In this mechanism, the large-scale magnetic field can be induced by a large-scale rotational motion in the presence of the cross correlation between the small-scale velocity and magnetic field. One problem, however, is the generation of significant cross-helicity. In order that $|\boldsymbol{u}' \cdot \boldsymbol{b}'|$ becomes large, $\boldsymbol{u}'$ and $\boldsymbol{b}'$ should have parallel (or antiparallel) components. Alfvén waves have this property. However, there has to be a mechanism that selects Alfvén waves travelling parallel to the magnetic field from those travelling antiparallel to the field. Furthermore, the small scale magnetic field has to be strong enough. In that sense this mechanism resembles that of Poezd et al. (1993) and Beck et al. (1995), where a strong small scale magnetic field was assumed to be generated by a small scale dynamo. The purpose of this Letter is to point out that, although the growth of the field by the cross-helicity effect is linear in time, fields of appreciable strength can be generated much earlier than by conventional dynamos, where the field grows exponentially from some seed magnetic field. Before we consider this problem more quantitatively, we begin by briefly sketching the nature of the cross-helicity effect.

2. Phenomenology and magnitude of the cross-helicity effect

As in all mean field theories one is interested in expressing the electromotive force,

$$\boldsymbol{\mathcal{E}} = \langle \boldsymbol{u}' \times \boldsymbol{b}' \rangle, \tag{1}$$

resulting from the small scales, in terms of large scale quantities. Normally, those large scale quantities include the mean magnetic field $\boldsymbol{B}$ and the mean current $\boldsymbol{J} = c\nabla \times \boldsymbol{B}/4\pi$. However, Yoshizawa (1990) showed that also the mean vorticity $\nabla \times \boldsymbol{U}$ enters this equation, where $\boldsymbol{U}$ is the mean velocity. A

z-component of the electromotive force would directly generate toroidal magnetic field, unlike the $\alpha\Omega$-dynamo, where toroidal field is generated from poloidal by differential rotation.

The effect of a mean vorticity is caused by the inertial term in the momentum equation and the stretching term in the induction equation (see Yokoi 1996). To leading order of this effect, the momentum and induction equations for the fluctuations, $\boldsymbol{u}'$ and $\boldsymbol{b}'$, read

$$\frac{\partial \boldsymbol{u}'}{\partial t} = -(\boldsymbol{u}' \cdot \nabla)\boldsymbol{U} + ..., \tag{2}$$

$$\frac{\partial \boldsymbol{b}'}{\partial t} = +(\boldsymbol{b}' \cdot \nabla)\boldsymbol{U} + ..., \tag{3}$$

where the dots refer to further terms that have been ignored for the purpose of the present illustration. Taking the cross product of those equations with $\boldsymbol{b}'$ and $\boldsymbol{u}'$, respectively, we obtain

$$\frac{\partial}{\partial t}(\boldsymbol{u}' \times \boldsymbol{b}') = \boldsymbol{b}' \times (\boldsymbol{u}' \cdot \nabla)\boldsymbol{U} + \boldsymbol{u}' \times (\boldsymbol{b}' \cdot \nabla)\boldsymbol{U} + ... \tag{4}$$

We assume isotropic turbulence with $\langle u_i' b_k' \rangle = \frac{1}{3}\delta_{ik}\langle \boldsymbol{u}' \cdot \boldsymbol{b}' \rangle$, replace the integration after averaging by a multiplication with some correlation (or turnover) time τ, and obtain

$$\boldsymbol{\mathcal{E}} = \tfrac{2}{3}\tau\langle \boldsymbol{u}' \cdot \boldsymbol{b}' \rangle \nabla \times \boldsymbol{U} + ... \tag{5}$$

Note that the electromotive force associated with a non-vanishing cross-helicity, $\langle \boldsymbol{u}' \cdot \boldsymbol{b}' \rangle \neq 0$, gives a non-zero contribution to the field generation even if $\Omega = \text{const}$.

In order to get some idea of how important this effect could be we now take a look at three different data sets of three-dimensional turbulence simulations. We first use data of stratified convection of Brandenburg et al. (1996). In this simulation an initially weak magnetic field is amplified exponentially by dynamo action until saturation occurs. The magnetic field has no large scale component, so we refer to this dynamo as a small-scale dynamo. However, this (local) simulation has been carried out at $30°$ northern latitude, so the turbulence has net helicity. The resulting value of the relative cross helicity, $\langle \boldsymbol{u}' \cdot \boldsymbol{b}' \rangle/(u_t b_t)$, where u_t and b_t are the root-mean-square values of the velocity and magnetic field, respectively, is around 0.03.

Simulations of rotating shear flow turbulence (Brandenburg et al. 1995) give a lower value for cross-helicity. Here the turbulence is the result of a magnetic shearing (or Balbus-Hawley) instability, where the magnetic field, in turn, is generated by dynamo action. In this case the resulting relative cross-helicity is 3×10^{-4}.

Simulations of the turbulence driven primarily by supernova explosions are perhaps more directly relevant to galaxies (Korpi et al. 1998). The resulting value of the relative cross-helicity is 5×10^{-3}. However, the simulations have not yet been run for long enough (just 5 Myr) to allow for the development of a statistically steady state. This may also be the reason why the sign of the cross-helicity is the same on both sides of the equatorial plane. In the first two cases the sign of $\langle \boldsymbol{u}' \cdot \boldsymbol{b}' \rangle$ is positive above the equatorial plane and negative below.

In conclusion, based on a variety of different simulations we expect the relative cross-helicity to be in the range $3 \times 10^{-2 \ldots -4}$. In the following we shall see that even for the smallest value the effect is large enough to cause an appreciable magnetic field after a few Gyr.

3. Application to galaxies

In order to generate cross-helicity there must be small scale dynamo action. This is typically a fast process occurring on the time scale of a few turnover times. If the growth rate of the small-scale dynamo is exactly one turnover time the amplification factor after 30 turnover times would be $e^{30} \approx 10^{13}$.

Let us now consider in more detail how the cross-helicity produces large-scale magnetic fields. Taking into account the standard α-effect, turbulent diffusivity and cross-helicity effect, the induction equation for the mean magnetic field can be written in the form

$$\begin{aligned}\frac{\partial \boldsymbol{B}}{\partial t} &= \nabla \times (\boldsymbol{U} \times \boldsymbol{B}) + \nabla \times (\alpha \boldsymbol{B}) \\ &\quad -\nabla \times (\eta \nabla \times \boldsymbol{B}) + \boldsymbol{S},\end{aligned} \tag{6}$$

where η is the turbulent diffusivity and the source term, $\boldsymbol{S} = \nabla \times (\lambda \nabla \times \boldsymbol{U})$, is caused by cross-helicity, $\lambda = \frac{2}{3}\tau\langle \boldsymbol{u}' \cdot \boldsymbol{b}' \rangle$. The source term can be written as

$$\boldsymbol{S} = \lambda \nabla \times (\nabla \times \boldsymbol{U}) - (\nabla \times \boldsymbol{U}) \times \nabla\lambda. \tag{7}$$

If the mean motion is rotation, i.e. $\boldsymbol{U} = s\Omega \boldsymbol{e}_\varphi$ with s being the cylindrical radius, then

$$\begin{aligned}\boldsymbol{S} = \frac{\lambda}{s^2}\left[\nabla\left(s^2 \frac{\partial \Omega}{\partial \varphi}\right) - s^3 \boldsymbol{e}_\varphi \left(\Delta\Omega + \frac{2}{s}\frac{\partial \Omega}{\partial s}\right)\right] \\ -\frac{1}{s}\,\boldsymbol{e}_\varphi \nabla(s^2\Omega) \cdot \nabla\lambda + \frac{1}{s^2}\,\frac{\partial \lambda}{\partial \varphi}\,\nabla(s^2\Omega)\,.\end{aligned} \tag{8}$$

As mentioned above, the cross-helicity dynamo produces only toroidal magnetic field, provided the mean flow and λ are axisymmetric. However, if Ω or λ are nonaxisymmetric this dynamo can directly generate a poloidal magnetic field, even if the α-effect is negligible. Evidently, in the case of rigid rotation, $\nabla \times \boldsymbol{U} = 2\boldsymbol{\Omega}$ =const and cross-helicity gives a non-zero contribution only if λ is nonuniform. The effect of cross-helicity is vanishing for rotation that is constant on cylinders with $\Omega \propto s^{-2}$. Note that Yokoi (1996) proposed the solution $\boldsymbol{B} = (\lambda/\eta)\boldsymbol{U}$. However, this cannot be correct, because it predicts nonvanishing magnetic fields for $\Omega \propto s^{-2}$ as well as for potential motions with $\nabla \times \boldsymbol{U} = 0$.

Let us now estimate the rate at which the cross-helicity produces large-scale magnetic fields. Note that contrary to the conventional α-dynamo, the cross-helicity dynamo does not require a non-vanishing initial magnetic field thus the growth of a large-scale field can start even if the seed field is zero. Obviously, during the initial stage of generation, when the magnetic field is weak, the growth is completely determined by cross-helicity and follows approximately a linear law,

$$\boldsymbol{B} \approx t\boldsymbol{S}. \tag{9}$$

In the course of further evolution the α-effect and turbulent diffusion also become important. For the purpose of illustration let us neglect in Eq. (6) the induction term caused by differential rotation. Consider the case when both rotation and turbulence are axisymmetric and the rotation law differs from $\Omega \propto s^{-2}$, which is likely to be the case for young galaxies. The effects of α-generation and turbulent diffusivity can roughly be modelled by corresponding inverse timescales γ_α and γ_η, respectively, thus the model equation can be written in the form

$$\dot{B} = (\gamma_\alpha - \gamma_\eta)B + S. \tag{10}$$

The general solution of this equation is

$$B = \frac{S}{\gamma_\alpha - \gamma_\eta}\left[e^{(\gamma_\alpha-\gamma_\eta)t} - 1\right] + B_0 e^{(\gamma_\alpha-\gamma_\eta)t}, \tag{11}$$

where B_0 is the initial field. The first term on the r.h.s. describes the effect of cross-helicity combined with α-generation and turbulent diffusivity, the second term corresponds to the standard α-dynamo. Let us assume that $B_0 = 0$, so that the α-dynamo in its standard form does not work. Then the behaviour of the magnetic field is essentially determined by the dynamo parameter, $C = \gamma_\alpha/\gamma_\eta$. If $C > 1$, the magnetic field can grow exponentially with the growth rate typical for the standard dynamo. Although the initial field may be zero, a significant mean magnetic field is generated by the cross-helicity effect, before the exponential growth becomes important. The field amplification can then only be stopped by nonlinear effects. If $C < 1$, after the initial growth, which lasts $\sim (\gamma_\alpha - \gamma_\eta)^{-1}$, the field reaches a saturation value depending on the magnitudes of cross-helicity and turbulent diffusivity,

$$B_s \approx S(\gamma_\eta - \gamma_\alpha)^{-1} \approx 3\times10^{-2\ldots-4}\ (H^2/\eta)\ (\Omega\tau/s)\ u_t b_t, \tag{12}$$

where we suppose $\langle \boldsymbol{u}' \cdot \boldsymbol{b}' \rangle = 3 \times 10^{-2\ldots-4} u_t b_t$, $\gamma_\eta = \eta/H^2$, and H is the semithickness of the disc. Detailed analysis of data on high-redshift damped Lyman-α systems which are widely believed to be the progenitors of current massive galaxies shows that models with discs that rotate rapidly and are thick give a better fit to observations (Prochaska & Wolfe 1997). The most likely values of the rotational velocity and thickness are 225 km/s and $H = 0.3s$, respectively. Assuming the characteristic length-scale of turbulence to be comparable to H, and $\eta = \frac{1}{3}u_t H$ we obtain for the saturation field

$$B_s \sim 3 \times 10^{-2\ldots-4}(\Omega\tau)b_t. \tag{13}$$

The turnover time τ in galactic discs is $\tau \sim 10^7$ yr, and so even for rapidly rotating discs $\Omega\tau \sim 0.1 - 0.3$. Thus, it seems that the cross-helicity effect alone may not be able to generate a large scale magnetic field in galaxies – in contradiction with the conclusion obtained by Yokoi (1996). However, the field given by Eq. (13) is only 2–4 orders of magnitude weaker than observed magnetic fields in galaxies. Therefore, as seen from Eq. (12), the α-effect may well be able to amplify the field produced by cross-helicity to the typical galactic value of a few μG after a few revolutions.

4. Numerical solutions

The estimate based on Eq. (13) is quite rough. Therefore we now consider an explicit model numerically. The method adopted is similar to that described by Brandenburg et al. (1993). We solve the dynamo Eq. (6) in a sphere $r \leq R$, where r is the spherical radius. Outside the sphere a vacuum is assumed, so the magnetic field continues as a potential field. Inside the sphere the profiles of α, λ, η, and Ω are given by

$$\frac{\alpha}{\alpha_0} = \frac{\lambda}{\lambda_0} = \frac{z}{H}\exp\left\{\frac{1}{2}\left[1 - \left(\frac{z}{H}\right)^2\right]\right\} + \epsilon, \tag{14}$$

$$\eta = \eta_{\rm h} + (\eta_{\rm d} - \eta_{\rm h})\exp\left[-\frac{1}{2}\left(\frac{z}{H}\right)^2\right], \tag{15}$$

$$\Omega = V_0\left(r_0^2 + r^2\right)^{-1/2}. \tag{16}$$

Here, ϵ quantifies a small perturbation of the otherwise purely antisymmetric profiles of α and λ; see below. The magnetic diffusivities in disc and halo are $\eta_{\rm d}$ and $\eta_{\rm h}$, respectively. In most of the cases we included a nonlinear effect in the form of α-quenching, i.e. we replace α_0 by $\alpha_0/(1 + \boldsymbol{B}^2/B_{\rm eq}^2)$.

The parity of the magnetic field generated by the cross-helicity effect is odd, i.e. dipole-like. This is because λ is antisymmetric about the equator. On the other hand, the parity of the most easily excited mode of the $\alpha\Omega$ dynamo is even, i.e. quadrupole-like (Parker 1971). The only way that the parity can change is by some 'impurities' in the system that give rise to a transfer of energy between purely antisymmetric and purely symmetric modes. We have therefore introduced in Eq. (14) a term ϵ of even parity with

$$\epsilon = \epsilon_0 \exp\left[-\frac{1}{2}\left(\frac{z}{H}\right)^2\right], \tag{17}$$

where $\epsilon_0 \ll 1$ controls the magnitude of the symmetry-breaking effect.

We present the results in dimensional form assuming $R = 10\,{\rm kpc}$, $\eta_{\rm d} = 1\,{\rm kpc\,km/s}$. We assume the turbulent velocities in the halo to be larger than in the disc, so the turbulent magnetic diffusivity in the halo is enhanced and we assume $\eta_{\rm h} = 10\eta_{\rm d}$. We assume $H = 1\,{\rm kpc}$, $r_0 = 1.5\,{\rm kpc}$. We also tried larger values of H, but the nature of the dynamo changed then considerably (the fields became oscillatory and of odd parity). With the set of parameters chosen the critical value of α_0 for dynamo action (when $\lambda_0 = 0$) is 0.48 km/s. We take $B_{\rm eq} = 10^{-6}$ G, which results in a saturation value of the maximum field in the disc of a few μG. Assuming a relative cross-helicity of $3 \times 10^{-2\ldots-4}$, we find $\lambda_0 = \frac{2}{3}\tau \times 3 \times 10^{-2\ldots-4}\, u_t b_t$, or

$$\lambda_0 = 2 \times 10^{-9\ldots-11}\,(10\,{\rm Gyr\,G\,km/s}), \tag{18}$$

where we have assumed $\tau = 10^7$ yr, $u_t = 10$ km/s, and $b_t = 10^{-5}$ G. In those cases where α_0 is non-vanishing we chose an approximately ten times supercritical value, $\alpha_0 = 5$ km/s.

We start off the calculation with a weak initial magnetic field of about 10^{-18} G, but note that this initial field is completely unimportant in all cases, except when $\lambda_0 = 0$. We show in Fig. 1 the evolution of the magnetic field B (the maximum value at any given time) and the field parity $P = [E^{(S)} - E^{(A)}]/[E^{(S)} + E^{(A)}]$, where $E^{(S)}$ and $E^{(A)}$ are respectively the energies in the

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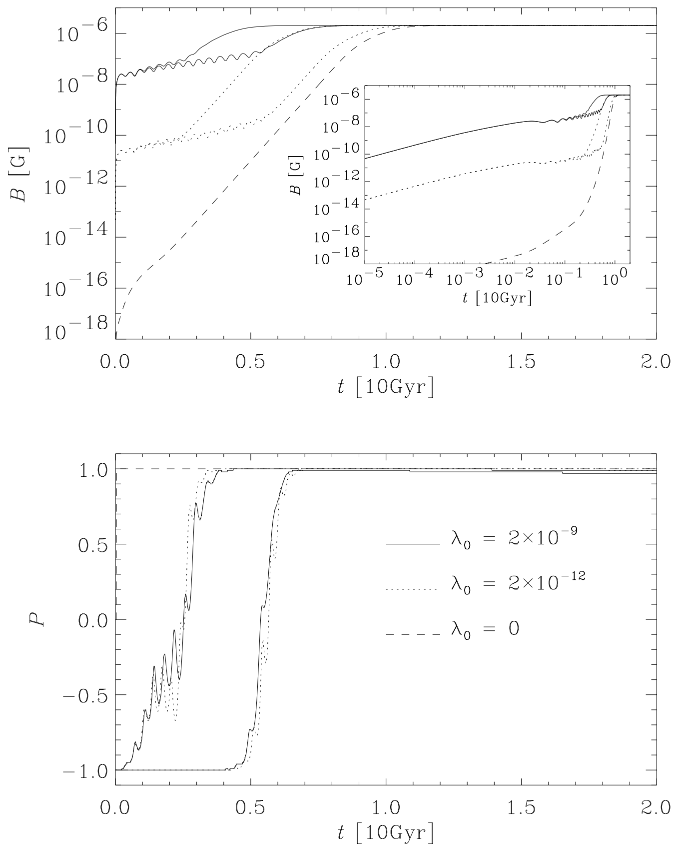

Fig. 1. Magnetic field B (upper panel) and magnetic parity (lower panel) as a function of time for four different models. The solid and dotted lines are for $\lambda_0 = 2 \times 10^{-9}$ and 2×10^{-12}, respectively. For both line types the upper and lower curves are for $\epsilon_0 = 10^{-2}$ and 10^{-5}, respectively. The dashed line gives the result for a pure $\alpha\Omega$-dynamo with $\lambda_0 = 0$. Note that all curves start with $B = 10^{-18}$ G, but in those cases where $\lambda_0 \neq 0$ the cross-helicity effect leads to a linear growth of the field to $\approx 10^{-8}$ G within approximately three turnover times (see inset of first panel).

symmetric and antisymmetric components of the field. ($P = +1$ for even, quadrupole-like fields and -1 for odd, dipole-like fields; see e.g. Brandenburg et al. 1992) During early stages of the evolution the growth is linear due to the cross-helicity effect and after ~ 0.1 Gyr the field reaches a relatively high value $\sim 0.01\mu$G. Later the α-effect accelerates the growth and around the time $t = 3 - 5$ Gyr the growth has become exponential. However, the exponential growth lasts only a rather short time of about $2 - 4$ Gyr, before the field reaches saturation.

The time when the field parity changes from odd to even is between 3 and 7 Gyr, depending on the value of ϵ_0. After the field parity has switched from -1 to $+1$, the magnetic field strength has increased by a factor of 10 to a few microgauss. In the absence of the cross-helicity effect the $\alpha\Omega$ dynamo leads to exponential growth starting from a weak seed magnetic field. In that case the field strength at a time of 3 Gyr is still only $10^{-14}\,\mu$G – too weak to explain the magnetic field observed in young high redshift galaxies.

5. Conclusion

We have shown that in the presence of the cross-helicity effect the large scale field reaches equipartition field strengths much sooner (after $4 - 6$ Gyr) than with a conventional $\alpha\Omega$-dynamo. The growth of the large scale magnetic fields is significantly enhanced. The presence of cross-helicity requires the presence of a small scale magnetic field that correlates with the velocity in such a way that the two are either preferentially parallel or antiparallel. Simulations show that this may indeed the case, although the magnitude of the effect is still uncertain. More work is needed to establish the existence and significance of the cross-helicity effect in realistic settings.

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